Learning outcomes:

- Set up and solve LP problems with simplex tableau.
- Interpret the meaning of every number in a simplex tableau.

Dear students, today we discuss small cases or case-lets on the above topic. The situations presented below are real business problems, modified slightly to suit our purpose.

We start now.

Case-let-1

A firm makes air coolers of three types and markets these under the brand name “Symphony”.

The relevant details are as follows:

<table>
<thead>
<tr>
<th>Profit / unit (Rs.)</th>
<th>300 Product A (hrs. / unit)</th>
<th>700 Product B (hrs. / unit)</th>
<th>900 Product C (hrs./ unit)</th>
<th>Total hrs. available</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designing</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>320</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>60</td>
<td>90</td>
<td>120</td>
<td>1600</td>
</tr>
<tr>
<td>Painting</td>
<td>30</td>
<td>40</td>
<td>60</td>
<td>1120</td>
</tr>
</tbody>
</table>

What Qty. of each product must be made to maximize the total profit of the firm?

Solution

Let $X_1, X_2, X_3$ denote the Qty. of each product made

Then: Maximize: $300 X_1 + 700 X_2 + 900 X_3$

Subject to: $0 X_1 + 10 X_2 + 20 X_3 \leq 320$

$60 X_1 + 90 X_2 + 120 X_3 \leq 1600$
Introducing slacks:
Maximize: \[ 300X_1 + 700X_2 + 900X_3 + 0S_1 + 0S_2 + 0S_3 \]
Subject to:
\[ \begin{align*} 
0X_1 + 10X_2 + 20X_3 + S_1 + 0S_2 + 0S_3 &= 320 \\
60X_1 + 90X_2 + 120X_3 + 0S_1 + S_2 + 0S_3 &= 1600 \\
30X_1 + 40X_2 + 60X_3 + 0S_1 + 0S_2 + S_3 &= 1120 
\end{align*} \]

1\textsuperscript{st} Tableau

<table>
<thead>
<tr>
<th>$C_j$ (Rs.)</th>
<th>Basic Act. Qty.</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$S_1$</td>
<td>320</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$S_2$</td>
<td>1600</td>
<td>60</td>
<td>90</td>
<td>120</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$S_3$</td>
<td>1120</td>
<td>30</td>
<td>40</td>
<td>60</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Z_j$</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C_j - Z_j$</td>
<td></td>
<td>300</td>
<td>700</td>
<td>900</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Pivot column = $X_3$

Pivot row = $S_2$ 
\[
\begin{bmatrix}
S_1 : 320 \\
S_2 : 1600 \\
S_3 : 1120 \\
Z_j : 0
\end{bmatrix} = \begin{bmatrix}
\frac{20}{3} \\
\frac{160}{120} \\
\frac{1}{4} \\
0
\end{bmatrix} = \begin{bmatrix}
16, 16, 3, 0
\end{bmatrix}
\]

Pivot element = 120

Pivot row updating:
\[
\begin{align*}
\frac{1600}{120} &= \frac{40}{3}, \quad \frac{60}{120} = \frac{1}{2} \cdot \frac{90}{120} = \frac{3}{4}, \quad 1, 0, \frac{1}{120}, 0
\end{align*}
\]

Up-dating $S_1$ row:
\[
\begin{align*}
320 - \left( 20 \times \frac{40}{3} \right) &= \frac{160}{3} \\
0 - \left( 120 \times \frac{1}{2} \right) &= -10 \\
20 - \left( 20 \times \frac{3}{4} \right) &= -5 \\
1 - (20 \times 0) &= 5 \\
0 - \left( 20 \times \frac{1}{120} \right) &= -\frac{1}{6} \\
1 - (20 \times 0) &= 0
\end{align*}
\]

Updating $S_3$ row:
\[
\begin{align*}
\frac{1}{20} - \left( 60 \times \frac{40}{3} \right) &= 320 \\
30 - \left( 60 \times \frac{1}{2} \right) &= -0 \\
40 - \left( 60 \times \frac{3}{4} \right) &= -5 \\
0 - (60 \times 0) &= 0 \\
0 - \left( 60 \times \frac{1}{120} \right) &= -\frac{1}{2} \\
1 - (60 \times 0) &= 1
\end{align*}
\]
## 2nd Tableau

<table>
<thead>
<tr>
<th>Cj (Rs.) (Rs.)</th>
<th>Basic Act.</th>
<th>Qty.</th>
<th>300 x₁</th>
<th>700 x₂</th>
<th>900 x₃</th>
<th>0 S₁</th>
<th>0 S₂</th>
<th>0 S₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>S₁</td>
<td>160/3</td>
<td>-10</td>
<td>-5</td>
<td>0</td>
<td>1</td>
<td>1/6</td>
<td>0</td>
</tr>
<tr>
<td>900</td>
<td>X₃</td>
<td>40/3</td>
<td>½</td>
<td>¾</td>
<td>1</td>
<td>0</td>
<td>1/120</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>S₃</td>
<td>320</td>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>-1/2</td>
<td>1</td>
</tr>
<tr>
<td>2000</td>
<td>Zj (Rs.)</td>
<td>450</td>
<td>450</td>
<td>675</td>
<td>900</td>
<td>0</td>
<td>15/2</td>
<td>0</td>
</tr>
<tr>
<td>Cj-Zj (Rs.)</td>
<td></td>
<td>-150</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-15/2</td>
<td>0</td>
</tr>
</tbody>
</table>

Since, there is still one + ve term in row, pivoting is reqd.

Pivot – Column = x₂

Pivot – row =

\[ x₃ \left(\begin{array}{c}
S₁ \div \frac{160}{3} = -5 (\text{not considere3d}) \\
S₂ \div \frac{40}{3} = \frac{160}{9} \\
S₃ \div \frac{320}{5} (\text{notconsi.})
\end{array}\right) \]

Pivot – element = \(3/4\)

Pivot-row updating:

\[
\frac{40}{3} \div \frac{3}{4} = \frac{160}{9}, \quad \frac{1}{2} \div \frac{3}{4} = \frac{2}{3}, \quad 1,1 \div \frac{3}{4} = \frac{4}{3}, \quad 0,1 \div \frac{3}{4} = \frac{1}{90}, 0
\]

Updating S₁ row:

\[
\begin{align*}
\frac{160}{3} \div \left(-5 \times \frac{160}{9}\right) &= \frac{1280}{9} \\
-10 \div \left(-5 \times \frac{2}{3}\right) &= \frac{20}{3} \\
-5 \div \left(-5 \times x₁\right) &= 0 \\
0 \div \left(-5 \times \frac{4}{3}\right) &= \frac{20}{3} \\
1 \div \left(-5 \times 0\right) &= 1 \\
\frac{1}{6} \div \left(-5 \times \frac{1}{90}\right) &= \frac{-1}{9} \\
0 \div \left(-5 \times 0\right) &= 0
\end{align*}
\]

Updating S₃ row:

\[
\begin{align*}
320 \div \left(-5 \times \frac{160}{9}\right) &= \frac{3680}{9} \\
0 \div \left(-5 \times \frac{2}{3}\right) &= \frac{10}{3} \\
-5 \div \left(-5 \times x₁\right) &= 0 \\
0 \div \left(-5 \times \frac{4}{3}\right) &= \frac{20}{3} \\
0 \div \left(-5 \times 0\right) &= 0 \\
\frac{1}{6} \div \left(-5 \times \frac{1}{90}\right) &= \frac{-4}{9} \\
1 \div \left(-5 \times 0\right) &= 1
\end{align*}
\]
IIIrd Tableau

<table>
<thead>
<tr>
<th>Cj (Rs.) (Rs.)</th>
<th>Basic Act.</th>
<th>Qty.</th>
<th>300 (x_1)</th>
<th>700 (x_2)</th>
<th>900 (x_3)</th>
<th>0 (S_1)</th>
<th>0 (S_2)</th>
<th>0 (S_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(x_1)</td>
<td>1280/9</td>
<td>-20/30</td>
<td>20/3</td>
<td>1-</td>
<td>1/9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>700</td>
<td>(S_2)</td>
<td>160/9</td>
<td>2/3</td>
<td>4/3</td>
<td>0</td>
<td>1-90</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>(S_3)</td>
<td>3680/9</td>
<td>10/3</td>
<td>20/3</td>
<td>0</td>
<td>-4/9</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>Zj (Rs.)</td>
<td>112000</td>
<td>1400/3</td>
<td>2800/3</td>
<td>0</td>
<td>70/9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>Cj-Zj (Rs.)</td>
<td></td>
<td>-500/3</td>
<td>-100/3</td>
<td>0</td>
<td>-70/9</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since Cj – Zj row has no. + ve value left.
∴ It is the optional soln.

Case-let-II

Two materials A & B are required to construct table & book cases. For one table, 12 units of A & 16 units of B are required while for a book case 16 units of A and 8 units of B are required. The profit on book case is Rs 25 and Rs 20 on a table. 100 units of A & 80 units of B are available. Formulate as a Linear programming problem & determine the optimal number of book cases & tables to be produced so as to maximise the profits.

Solution

Let \(x_1\) = no. of unit of tables to be produced, and \(X_2\) = no. of unit of book cases to be produced.

Formulating as a LP problem, we have:-

Maximise \(Z = 20x_1 + 25x_2\) (objective function)

Subject to the constraints: \(12x_1 + 16x_2 \leq 100\)
\(16x_1 + 8x_2 \leq 80\)
\(x_1, x_2 \geq 0\)

Introducing the slack variable, we have:
\(12x_1 + 16x_2 + S_1 \leq 100\)
\(16x_1 + 8x_2 + S_2 \leq 80\)
Re-writing as:
\[
\begin{bmatrix}
12 & 16 & 1 & 1 \\
16 & 8 & 0 & 100 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
S_1 \\
S_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
100 \\
80 \\
\end{bmatrix}
\]
or
\[P_0x_1 + P_2x_2 + P_3S_1 + P_4S_2 = P_0\]

\[\therefore \quad \text{our problem becomes Maximise} \]

\[F = 20x_1 + 25x_2 + 0 \times S_1 + 0 \times S_2 \quad \ldots (1)\]

Where \(P_0x_1 + P_3x_2 + P_3S_1 + P_4S_2 = P_0\) \ldots (ii)

element of \(C_j\) row are values of \(P_0, P_2, P_4, P_1\), in (i) by comparing with (2) i.e. elements of \(C_j\) are 0, 0, 20, 25.

**Simplex Method**

<table>
<thead>
<tr>
<th>(\text{Stage I})</th>
<th>(C_j)</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>20</th>
<th>25</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vectors (P_0, P_2, P_4, P_1, P_3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>(R_1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12</td>
<td>16</td>
<td>(100/16 = 6.25)</td>
</tr>
<tr>
<td>(R_2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16</td>
<td>0</td>
<td>(80/8 = 10)</td>
</tr>
<tr>
<td>(Z_j)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.25 is least ratio</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\begin{align*}
\text{Z}_j - & \quad C_j \\
& \begin{array}{l}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{array} \\
& \begin{array}{l}
20 \\
25 \\
\end{array} \\
\end{align*}\]

\[\therefore \quad \text{replaced vector is } P_2\]

\[\begin{align*}
\text{Stage I} \\
\text{Z}_j & = \text{least ratio} \\
\text{replaced vector is } P_2\]

\[\begin{align*}
\text{Stage I} & = \text{least ratio} \\
\text{replaced vector is } P_4\]

\[\begin{align*}
\text{Stage I} & = \text{least ratio} \\
\text{replaced vector is } P_1\]

The elements of column \(C_j\) are values of \(P_3\) and \(P_4\) is (i) as compared with (2). The column \(P_0, P_2, P_4, P_1, P_3\) are values of \(P_0, P_2, P_1\) etc. in (3)
Since in stage III all elements of row $Z_j - C_j$ are +ve or zero, hence an optimal solution has been achieved.

The solution is given by column is Stage III.

$$P_0 = 3P_1 + 4P_2$$

compare it with $P_1x_1 + P_2x_2 + P_3S_1 + P_4S_2 = P_0$

$$x_1 = 3, x_2 = 4, S_1 = 0, S_2 = 0$$

Note I. the elements of row $Z_j$ is the sum of product of corresponding elements of column $C_j$ with $P_0, P_3, P_4, P_1, P_2$ respectively.

For example in stage II, the elements of row $Z_j$ are

$$25x \frac{25}{4} + 0x \frac{30}{4} = \frac{625}{4}$$

$$25x \frac{1}{16} + 0x = \frac{\frac{-1}{2} \cdot 25}{16}$$

$$25x 0 + 0x = 0$$

$$25x \frac{3}{4} + 0x 10 = \frac{75}{4}$$

$$25x 1 + 0x 0 = 25$$

Note 2. When we replace vector $P_2$ in place of $P_3$ in stage II, the elements to left of $P_3$ under column $C_j$ is also changed.

Note. To check the results.

At $x = 3, x_2 = 4$

Constraints are

$$12x_1 + 16x_2 = 36 + 64 = 100 \leq 100$$

$$16x_1 + 8x_2 = 48 + 32 = 80 \leq 80$$

Case-let-III

A manufacturer produces children’s bicycles & scooters both of which one processed through two machines. Machine 1 has a maximum of 120 hours available & machine 2 has a maximum of 180 hours. Manufacturing a bicycle requires 6 hours on machine 1 x 4 hours on machine 2. A scooter requires 3 hours on machine 1 & ten hours on machine 2. If the profit is Rs 45 on a bicycle & Rs. 55 on a scooter determine the number of bicycles & scooters that should be produced in order to maximise profit.
Solution

Let no. of bicycles and scooters produced by x and y units respectively.
∴ L.P. problem is Maximize \( P = 45x + 55y \)
Subject to
\[
6x + 3y \leq 120 \\
4x + 10y \leq 180, \ x \geq 0, y \geq 0
\]

Introducing the slack variables, we get
\[
6x + 3y + S_1 + 120 \\
4x + 10y + S_2 + 180
\]
writing it in vector form
\[
\begin{bmatrix}
6 \\
4
\end{bmatrix}x + \begin{bmatrix}
3 \\
4
\end{bmatrix}y + \begin{bmatrix}
1 \\
0
\end{bmatrix}S_1 + \begin{bmatrix}
0 \\
1
\end{bmatrix}S_2 = \begin{bmatrix}
120 \\
180
\end{bmatrix}
\]

or \( P_1x + P_2y + p_3S_1 + P_4S_2 = P_0 \)
Θ Our L.P.P is Max, \( P=45x+55y+OS_1+OS_2 \)
Θ Subject to \( P_1x + P_2y + p_3S_1 + P_4S_2 = P_0 \) \( \text{...(2)} \)
Comparing (1) and (3), we have
\[
P_1 = \begin{bmatrix}
6 \\
4
\end{bmatrix}, \quad P_2 = \begin{bmatrix}
3 \\
10
\end{bmatrix}, \quad P_3 = \begin{bmatrix}
1 \\
0
\end{bmatrix}, \quad P_4 = \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

\[
P_0 = \begin{bmatrix}
120 \\
180
\end{bmatrix}
\]
The elements of row are value of
Po, P2,P4,P1,P3, in (1) by comparing it with (2)
i.e., element of row are 0, 0, 45,45.,55.

**Note.** The element of row are sum of product of corresponding element of columns and column vectors

<table>
<thead>
<tr>
<th>Simplex Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_j )</td>
</tr>
<tr>
<td>( \text{Vectors} )</td>
</tr>
<tr>
<td>( R_1 )</td>
</tr>
<tr>
<td>( R_2 )</td>
</tr>
<tr>
<td>( Z_j )</td>
</tr>
<tr>
<td>( Z_j-C_j )</td>
</tr>
<tr>
<td>( \Theta - 55 ) is least no. in row ( Z_j-C_j ) ( \therefore ) replacing vector is ( P_1 )</td>
</tr>
</tbody>
</table>

Row \( R_1 \) \( \frac{R_1}{a_{31}} = \frac{R_1}{6} \)
Row \( R_2 = R_2 - R_1 \times \frac{a_{41}}{a_{31}} \times R_2 - R_1 \times \frac{4}{6} \).

Further calculations are left to the students as an exercise.

**Case-let-IV**

A firm has the following availabilities:

<table>
<thead>
<tr>
<th>Type-available</th>
<th>Amount-available (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>240</td>
</tr>
<tr>
<td>Plastic</td>
<td>370</td>
</tr>
<tr>
<td>Steel</td>
<td>180</td>
</tr>
</tbody>
</table>

The firm produces two products A & B having a selling price of Rs. 4 per unit & Rs. 6 per unit respectively. The requirements for the manufacture of A & B are as follows:

<table>
<thead>
<tr>
<th>Product</th>
<th>Requirements of (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wood</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
</tr>
</tbody>
</table>

Formulate as a LP problem & solve by using the simplex method to maximise the gross income of the firm.

**Case-let-V**

Ace-advantage Ltd. faces the following situation:

**Media available** - electronic (A) & print (B)

**Cost of available in** - media A: Rs. 1000
                        - media B: Rs. 1500

**Annual advertising budget** - Rs. 20000

The following constraints are applicable:

Electronic Media (A) can not have more than 12 advertisements in a year and not less than 5 advertisement must be placed in the print media (B).

The estimated audience are as follows:

Electronic media (A) - 40000
Print media (B) - 55000

You are required to develop a mathematical model & solve it for maximizing the total effective audience.
Case-let-VI

Khalifa & sons sells two different books B1 & B2 at a profit margin of Rs. 7 and Rs. 5 per book respectively. B1 requires 5 units of raw material & B2 requires 1 units of raw material. The maximum availability of raw materials is limited to 15 units. To maintain the high quality of books, it is desired to follow the given quality constraint: $3x_1 + 7x_2 \geq 21$. Formulating

As a LP model determine the optimal solution.

Dear students, we have now reached the end of our discussion scheduled for today. See you all in the next lecture.
Bye.