Unit 4

DECISION ANALYSIS

Lesson 37

Learning objectives:

- **To learn how to use decision trees.**
- **To structure complex decision making problems.**
- **To analyze the above problems.**
- **To find out limitations & advantages of decision tree analysis.**

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Decision Theory and Decision Trees

Hello Students,

Decision trees are excellent tools for making financial or number based decisions where a lot of complex information needs to be taken into account. They provide an effective structure in which alternative decisions and the implications of taking those decisions can be laid down and evaluated. They also help you to form an accurate, balanced picture of the risks and rewards that can result from a particular choice.

What is a decision tree ?

Decision tree is a classifier in the form of a tree structure.

So far, the decision criteria are applicable to situations where we have to make a single decision. There is another technique for analyzing a decision situation. A ***decision tree*** is a schematic diagram consisting of nodes and branches. There are some advantages of using this technique:

1. It provides a pictorial representation of the sequential decision process. Each process requires one payoff table.
2. It is easier to compute the expected value (opportunity loss). We can do them directly on the tree diagram.
3. More than one decision maker can be easily involved with the decision processes.

There are only three rules governing the construction of a decision tree:

● = state-of-nature (event) node

■ = decision node

branches emanating from a decision node reflect the alternative decisions possible at that point.

If a decision situation requires a series of decisions, then a payoff table approach cannot accommodate the multiple layers of decision - making. A decision tree approach becomes the best method. However, a tree diagram will contain nothing but more squares (decision nodes). We can still apply the expected value criterion at each decision node from the right to the left of a tree diagram (*backward process*).

A decision tree can be used to classify an example by starting at the root of the tree and moving through it until an event node occurs, which provides the classification of the instance.

Decision tree induction is a typical inductive approach to learn knowledge on classification. The key requirements to do mining with decision trees are:

- *Attribute-value description*: object or case must be expressible in terms of a fixed collection of properties or attributes. This means that we need to discretize continuous attributes, or this must have been provided in the algorithm.
- *Predefined classes (target attribute values)*: The categories to which examples are to be assigned must have been established beforehand (supervised data).
- *Discrete classes*: A case does or does not belong to a particular class, and there must be more cases than classes.
- *Sufficient data*: Usually hundreds or even thousands of training cases.

How to Draw a Decision Tree?

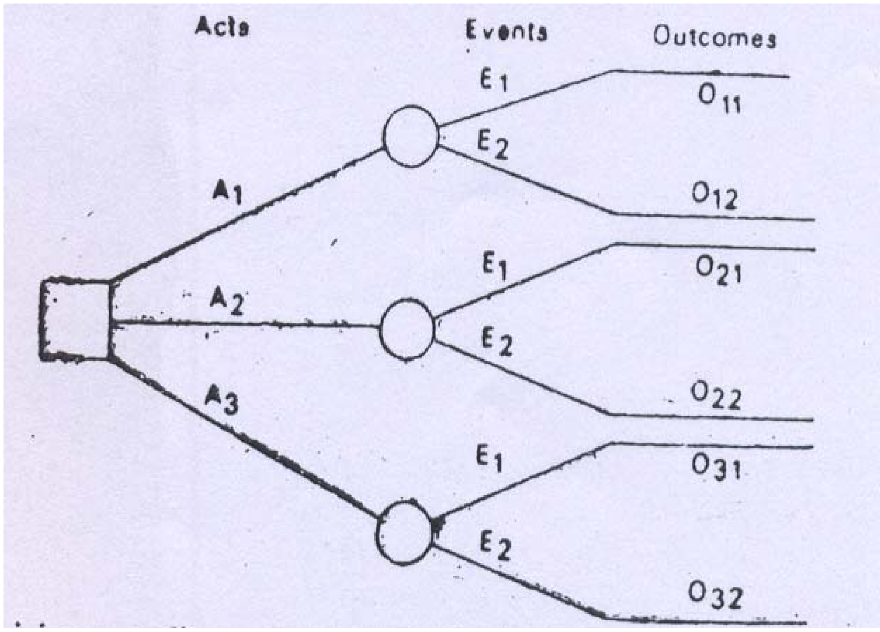
You start a decision tree with a decision that needs to be made. This decision is represented by a small square towards the left of a large piece of paper. From this box draw out lines towards the right for each possible solution, and write that solution along the line. Keep the lines apart as far as possible so that you can expand your thoughts.

At the end of each solution line, consider the results. If the result of taking that decision is uncertain, draw a small circle. If the result is another decision that needs to be made, draw another square. Squares represent decisions, circles represent uncertainty or random factors. Write the decision or factor to be considered above the square or circle. If you have completed the solution at the end of the line, just leave it blank.

Starting from the new decision squares on your diagram, draw out lines representing the options that could be taken. From the circles draw out lines representing possible outcomes. Again mark a brief note on the line saying what it means. Keep on doing this until you have drawn down as many of the possible outcomes and decisions as you can see leading on from your original decision.

An example of the sort of thing you will end up with is shown below:

PORTRAY OF A DECISION TREE



Once you have done this, review your tree diagram. Challenge each square and circle to see if there are any solutions or outcomes you have not considered. If there are, draw them in. If necessary, redraft your tree if parts of it are too congested or untidy.

You should now have a good understanding of the range of possible outcomes.

Starting to Evaluate Your Decision Tree

Now you are ready to evaluate the decision tree. This is where you can calculate the decision that has the greatest worth to you. Start by assigning a cash or numeric value to each possible outcome - how much you think it would be worth to you.

Next look at each circle (representing an uncertainty point) and estimate the probability of each outcome. If you use percentages, the total must come to 100% at each circle. If you use fractions, these must add up to 1. If you have data on past events you may be able to make rigorous estimates of the probabilities. Otherwise write down your best guess.

Calculating Tree Values

Once you have worked out the value of the outcomes, and have assessed the probability of the outcomes of uncertainty, it is time to start calculating the values that will help you make your decision.
We start on the right hand side of the decision tree, and work back towards the left. As we complete a set of calculations on a node (decision square or uncertainty circle), all we need to do is to record the result. All the calculations that lead to that result can be ignored from now on - effectively that branch of the tree can be discarded. This is called 'pruning the tree'.

Calculating The Value of Decision Nodes

When you are evaluating a decision node, write down the cost of each option along each decision line. Then subtract the cost from the value of that outcome that you have already calculated. This will give you a value that represents the benefit of that decision.
Sunk costs, amounts already spent, do not count for this analysis.
When you have calculated the benefit of each decision, select the decision that has the largest benefit, and take that as the decision made and the value of that node.

Now, consider this with a simple example…..

Example 1. An executive has to make a decision. He has four alternatives D1, D2, D3 and D4. When the decision has been made events may lead such that any of the four results may occur. The results are R1, R2, R3 and R4. Probabilities of occurrence of these results are as follows:

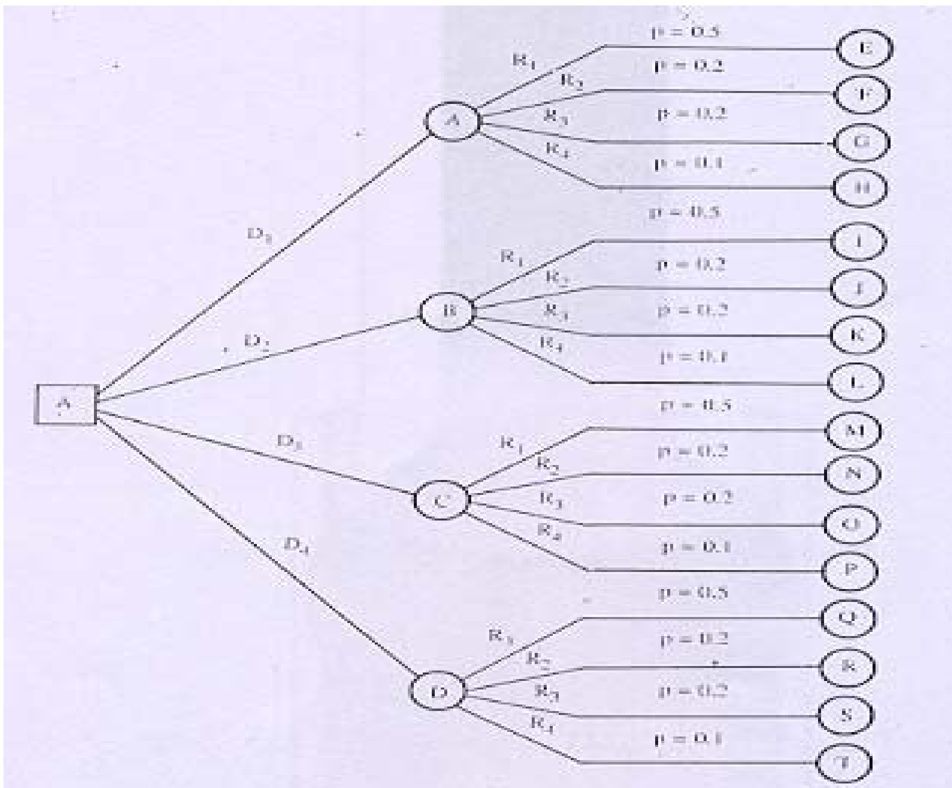
R1 = 0.5, R2 = 0.2, R3 = 0.2, R4 = 0.1

The matrix of pay-off between the decision and the results is indicated below:

	R1	R2	R3	R4
D1	14	9	10	5
D2	11	10	8	7
D3	9	10	10	11
D4	8	10	11	13

Show this decision situation in the form of a decision tree and indicate the most preferred decision and corresponding expected value.

Solution. A decision tree, which represents possible courses of action and states of nature are shown in the following figure. In order to analyses the tree, we start working backward from the end branches.



The most preferred decision at the decision node 1 found is by calculating expected value of each decision branch and selecting the path (course of action) with high value.

The expected monetary value of node A, B and C is calculate as follows:

EMV (A) = 0.5 x 14 + 0.2 x 9 + 0.2 x 10 + 0.1 x 5 = 11.3

EMV (B) = 0.5 x 11 + 0.2 x 10 + 0.2 x 8 + 0.1 x 7 = 9.8

EMV (C) = 0.5 x 9 + 0.2 x 10 +0.2 x 10 + 0.1 x 11 = 9.6

EMV (D) = 0.5 x 8 + 0.2 x 10 + 0.2 x 11 + 0.1 x 13 = 9.5

Since node A has the highest EMV, the decision at node 1 will be to choose the course of action D4.

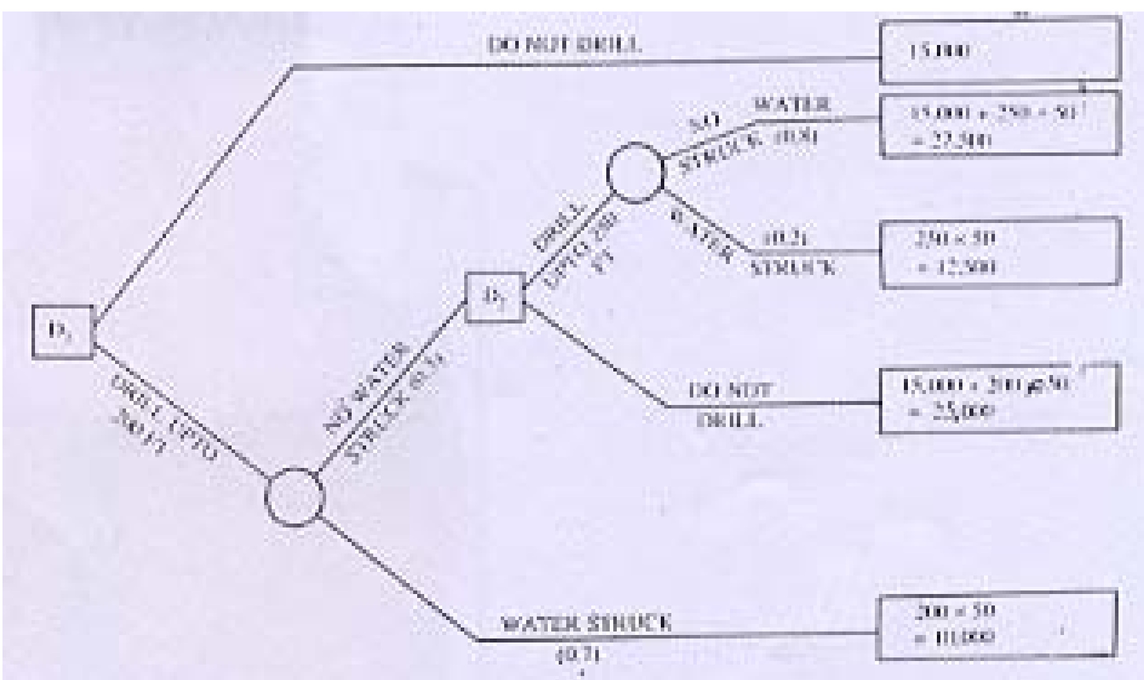
Example 2. A farm owner is seriously considering of drilling farm well. In the past, only 70% of wells drilled were successful at 200 feet of depth in the area. Moreover, on finding no water at 200 ft., some persons drilled it further up to 250 feet but only 20% struck water at 250 ft. The prevailing cost of drilling is Rs. 50 per feet. The farm owner has estimated that in case he loss not get his own well, he will have to pay Rs. 15,000 over the next 10 years (in PV terms) to buy water from the neighbor. The following decisions can be optimal:

(i) do not drill any well, (ii) drill up to 200 ft. (iii) if no water is found at 200 ft., drill further up to 250 ft.

Draw an appropriate decision tree and determine the farm owner's strategy under EMV approach.

Solution. The given data can easily be represented by the following decision tree diagram.

Consequence of outflow (Rs.)



There are two decision points in the tree indicated by 1 and 2. In order to decide between the two basis alternatives, we have to fold back (backward induction) the tree from the decision point 2, using EMV as criterion:

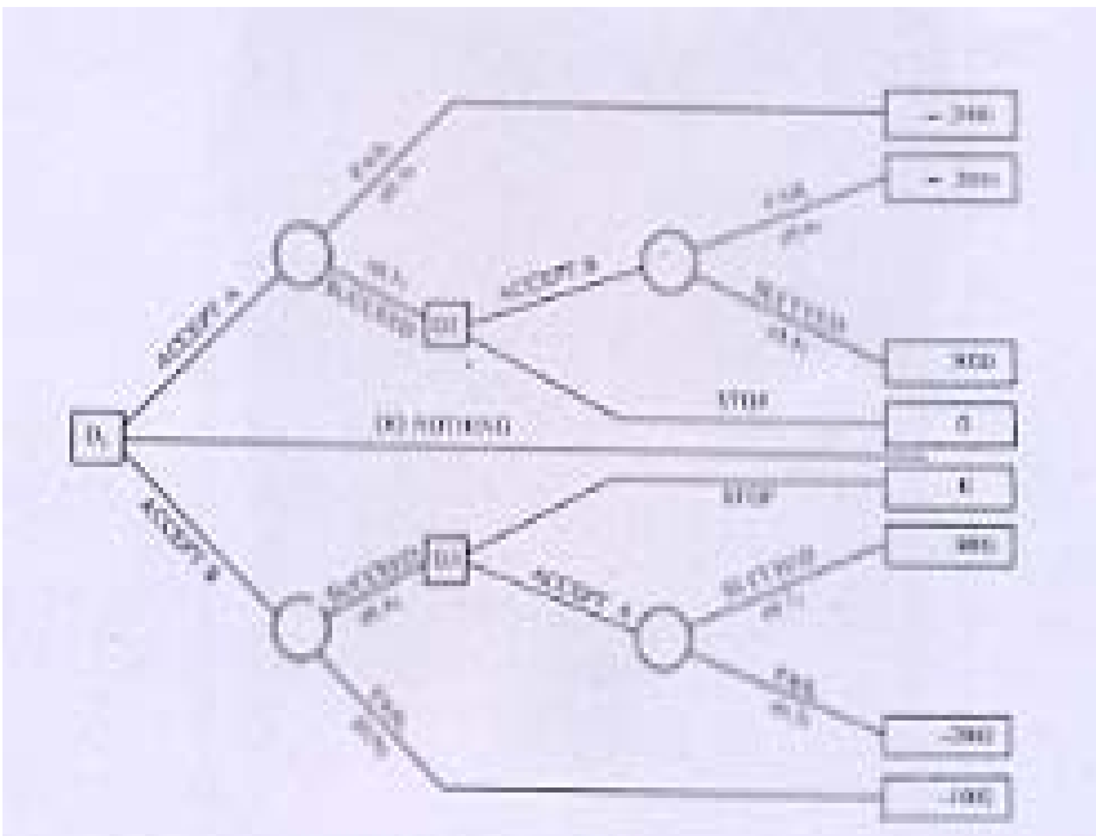
EVALUATION OF DECISION POINTS

Decision point	State of Nature	Probability	Cash outflows	Expected cash Outflow
		Decision at point D_2		
1. Drill up to 250 fit.	Water struck	0.2	Rs. 12,5000	Rs. 2,500
	No water struck	0.8	27,500	22,000
			EMV (outflows) = 24,500	
2. Do not drill up to 250 ft.		EMV (outflow) = Rs. 25,000		
	The decision at D_2 is : Drill up to 250 feet.			
		Decision at point D_2		
1. Drill up to 200 ft.	Water struck	0.7	Rs. 10,000	Rs. 7,000
	Not water struck	0.3	24,500	7,350
			EMV (outflow) = Rs. 14,350	
2. Do not drill up to 200 ft.		EMV (outflow) = Rs. 15,000		
	The decision at D_1 is : Drill up to 200 ft.			

Thus the optimal strategy for the farm-owner is to drill the well up to 200 ft. and if no water is struck, then further drill it up to 250 ft.

Example 3. A businessman has two independent investments A and B available to him; but he lacks the capital to undertake both of them simultaneously. He can choose to take A first and then stop, or if A is successful then take B, or vice versa. The probability of success of A is 0.7, while for B it is 0’4. Both investments require an initial capital outlay of Rs. 2,000, and both return nothing if the venture is unsuccessful. Successful competitions of A will return Rs. 3,000 (over cost), and successful completion of B will return Rs. 5,000 (over cost). Draw the decision tree and determine the best strategy.

Solution. The appropriate decision tree is shown below:



There are three decision points in the above decision tree indicated by D_1, D_2 and D_3.

EVALUATION OF DECISION POINTS

	Decision point	Outcome	Probability	Conditional Values	Expected Values
D_3	(i) Accept A	Success	0.7	Rs. 3000	Rs. 2100
		Failure	0.3	-Rs. 2000	-Rs. 600
					Rs. 1500
	(ii)	Stop			0
D_2	(i) Accept B	Success	0.4	Rs. 5000	Rs. 2000
		Failure	0.6	-Rs. 2000	-Rs. 1200
					Rs. 800
		Stop			0
D_1	(i) Accept A	Success	0.7	Rs. 3000 + 800	Rs. 2660
		Failure	0.3	-Rs. 2000	-Rs. 600
	(ii) Accept B	Success	0.4	Rs. 5000 + 1500	Rs. 2600
		Failure	0.6	-Rs. 2000	-Rs. 1200
					Rs. 1400
	(iii) Do Nothing				0

Hence, the best strategy is to accept A first, and if it is successful, then accept B.

Strengths and Weakness of Decision Tree Methods

The strengths of decision tree methods are:

- Decision trees are able to generate understandable rules.
- Decision trees perform classification without requiring much computation.
- Decision trees are able to handle both continuous and categorical variables.
- Decision trees provide a clear indication of which fields are most important for prediction or classification.

The weaknesses of decision tree methods

- Decision trees are less appropriate for estimation tasks where the goal is to predict the value of a continuous attribute.
- Decision trees are prone to errors in classification problems with many class and relatively small number of training examples.

- Decision tree can be computationally expensive to train. The process of growing a decision tree is computationally expensive. At each node, each candidate splitting field must be sorted before its best split can be found. In some algorithms, combinations of fields are used and a search must be made for optimal combining weights. Pruning algorithms can also be expensive since many candidate sub-trees must be formed and compared.
- Decision trees do not treat well non-rectangular regions. Most decision-tree algorithms only examine a single field at a time. This leads to rectangular classification boxes that may not correspond well with the actual distribution of records in the decision space.

In this lecture you must have got a hang as to how decision tree analysis is used to arrive at business decisions.

Summary

Decision trees provide an effective method of decision making because they:

- clearly lay out the problem so that all choices can be viewed, discussed and challenged
- provide a framework to quantify of the values of outcomes and the probabilities of achieving them
- help us to make the best decisions on the basis of our existing information and best guesses.

As with all decision-making methods, though, decision tree analysis should be used in conjunction with common sense. They are just one important part of your decision-making tool kit.