# Unit 4 <br> DECISION ANALYSIS 

## Lesson 36

## Learning Objective:

- Review risk as a decision environment, and review methods useful for making decisions in this environment.
- Demonstrate how monetary value and probability information can be combined for more effective decision making.
- Illustrate expected value as a decision criterion under conditions of risk.


## Decision Making Under Risk

In this situation, the decision maker faces several states of nature. But he is supposed to have believable evidential information, knowledge, experience or judgment to enable him to assign probability values to the likelihood of occurrence of each state of nature. Probabilities could be assigned to future events by reference to similar previous experience and information. Sometimes past experience, or past records often enable the decision-maker to assign probability values to the likely possible occurrence of each state of nature. Knowing the probability distribution of the states of nature, the best decision is to select that course of action that has the largest expected payoff value.

For decision problems involving risk situations one most popular method of decision criterion for evaluating the alternative strategies is the Expected Monetary Value (EMV) or expected pay-off. The objective should be to optimize the expected pay-off, which may mean either maximization of expected profit or minimization of expected regret.

Before this, let us have a look at certain conceptual approaches to Probability:

## The Concept of Probability

Probability theory is the rational way to think about uncertainty. It is the branch of mathematics devoted to measuring quantitatively the likelihood that a given event will occur. These two definitions derive from two different approaches to the concept of probability: subjective versus objective.


The objective probability viewpoint posits that the likelihood that a particular event will occur is a property of the system under study, which is ultimately grounded on the physical laws bearing on the given system. The subjective impressions that an observer may have about the likelihood of occurrence of that event in no way affect the actual probability of occurrence. Put succinctly, probability resides in the object, not the subject. It is wholly independent of the observer's state of mind.

The subjective probability viewpoint argues that the likelihood that a particular event will occur is a measure of the belief of the observer of the system given his/her state of information at the time. It is meaningless to talk about "the actual probability of occurrence" of an event because such a conception is unknowable and impossible to define outside the observer's mental space. Put succinctly, probability resides in the subject, not the object. It is intrinsically bound to the observer's state of mind.

All this may sound a tad philosophical -which it is- yet is relevant for the development and understanding of expected value decision models. To see why, let's examine in more detail what "objectivity" entails.

Objective probability can be approached axiomatically or statistically. Axiomatic probability refers to the use of the mathematical theory of probability (axioms and theorems) along with the logical framework of the system being studied to derive quantitative measures of the likelihood of occurrence of particular events solely on the basis of theoretical and logical considerations. In all such cases, clearly defining the sample space of interest
is essential. An example of an axiomatic probability assessment is the statement «the probability of heads on a coin toss is $1 / 2$ » when based on the assumptions that the coin is fair, the toss is unbiased, and the sample space consists of two symmetrical event subspaces (either heads or tails must come up; coins stuck upright in a groove, for instance, are disqualified). No actual toss of the coin is required. Axiomatic probability relies instead on gedanken (thought) experiments.

Statistical probability, on the other hand, makes use of physical experiments (in addition to both the mathematical theory of probability and the experiment's logical framework) to assess the likelihood of occurrence of events by means of relative frequency of event outcomes. Thus, statistical probability is empirical in nature. An example of a statistical probability assessment is the statement «the probability of heads on a coin toss is $1 / 2$ » when based on the results of numerous trials of actual coin tosses conducted under identical conditions. Actual trials, however, need not and often do not square up to an even $50-50$. Recourse to probability theory is required to reconcile experimental discrepancies with axiomatic inferences.

It should be noted that the two objective approaches to probability follow to the letter two of the three epistemologically valid approaches to ascertaining knowledge: rigorous mathematical/logical reasoning and controlled empirical procedures. (We will explore the third epistemologically valid approach later on.) So why should one bother with subjective probability?

Because, unfortunately, real-world problems are not always amenable to the demanding conditions imposed by objective probability. Consider Wolfgang Cactus and Goldie Lockes, ACME's executive managers. In order to determine objectively the probability distribution for the states of nature in their decision problem, they would need to either know everything affecting the market for road runner traps (including such things as the state of the world economy at every point in time throughout the five-year period covered by the decision!) in order to properly define the sample space for the gedanken experiment, or conduct numerous trials with the new, improved road runner traps under all possible market conditions to assess the probabilities statistically. The former approach is impossible because the required information is simply not obtainable (nor digestible), while the latter is impossible because once the first trial is conducted with the new traps, the market reacts (competitors may enter the market, for instance) and conditions will forever be different. Yes, market trials may not be perfect but can be of value. We'll look into this shortly. The point is that subsequent experimental conditions are no longer identical to the initial trial, violating the tenets of statistical probability.

In the absence of reliable objective probabilities, subjective estimates are the best game in town, say some folks. Even when they are available, retort others. Prof. Ronald Howard of Stanford likes to elucidate this with a fine little story. This astronaut was being strapped to his seat in the cockpit when he asks the crew chief if the rocket is safe. "It's $99.9 \%$ safe," replies the chief. "Determined axiomatically by NASA's engineers." The astronaut glances outside, sees an identical rocket on the neighboring launch pad, and requests that it be launched as a test. After much arguing from Mission Control (rockets don't come cheap), they acquiesce and launch the other rocket. Suddenly, moments into the liftoff, the thing explodes in a fireball. Strictly speaking, since the probability of a safe launch was determined axiomatically, the two rocket launches are independent events and the astronaut's rocket is still $99.9 \%$ safe. "Yeah, right," said the astronaut as he walked away from his rocket.

When you're in the cockpit, the only probability that matters is your own.

## Assessing Probabilities Subjectively

We recall that the Laplace decision criterion began with the premise that to deal with uncertainty rationally, probability theory must be employed. This means that a probability distribution must be assigned to every set of uncertain states of nature in the decision problem. As we saw on the previous page, probabilities can be determined either objectively or subjectively. If reliable objective probabilities are available, they should ordinarily be used. If, on the contrary, no reliable objective probabilities are available, Laplace prescribes that subjective probabilities are assessed. (It is only because no probabilities had been posted on the decision matrix that Laplace concluded, by the Principle of Insufficient Reason, that the states of nature had to be equally probable.) One way of putting it: it is better to have subjective probabilities, even if somewhat inaccurate, than to have no probabilities at all. For without probabilities, all decision criteria are less than satisfactory, as we have seen. Moreover, it is possible to revise subjective probabilities with access to additional information, thus improving the accuracy of the subjective estimates. That precisely is why market studies are performed.

Decision makers do not work in a vacuum. They usually know something, oftentimes quite a lot, about the decision problem they are dealing with including its environment and, hence, the states of nature affecting the decision. They routinely make use of this knowledge when managing their affairs. Consequently, quantifying their knowledge (and intuition) about the likelihood of occurrence of uncertain events is not at all unreasonable. In fact, it is the logical thing to do.

## A Method for Eliciting Subjective Probabilities:

1. Rank orders the states of nature $S_{j}$ in terms of their likelihood of occurrence. (Ties are allowed and should be denoted by placing tied states [ $S_{k}$, $S_{l}$, etc.] at the same level in the list.)
2. Assign an arbitrary weight of 1 (actually, any number will do) to the most likely event $S_{j}$.
3. Assess the degree of relative likelihood of the next state $S_{k}$ by assigning a fractional weight in proportion to the most likely state $S_{j}$. (Ties require a duplicate weight.)
4. Assess the degree of relative likelihood of the remaining states $S_{l}$ by assigning fractional weights in proportion to any other previously weighted state $S_{x}$.
5. Sum the weights.
6. Normalize the weights (divide each weight by the sum of the weights).
7. The resulting numbers are the probabilities of occurrence for each of the states. (They must add up to 1.)

Rationality is bounded, and people rarely possess the ability to recite a nontrivial probability distribution off the cuff. But it has been shown that pair wise comparisons between uncertain events lead to reasonably accurate probability estimates when the assessor is more or less informed about the problem at hand.

Let's assume that ACME's managers believe that the most likely market demand for newfangled road runner traps is $M$ (medium demand), followed by $W$ (low) and lastly, $H$ (high). The event list would be ordered accordingly. Assign a weight of 1 to event $M$.

| $\frac{\text { Event }}{M}$ | Weight |
| :---: | ---: |
| $W$ | 1 |
| $H$ |  |

Now suppose Cactus and Lockes believe $W$ to be half as likely as $M$, and $H$ to be one-third as likely as $W$.

| $\frac{\text { Event }}{M}$ | $\frac{\text { Weight }}{1}$ | $=6 / 6$ |
| :---: | :---: | :---: |
| $W$ | $1 / 2(1)$ | $=3 / 6$ |
| $H$ | $1 / 3(1 / 2)$ | $=\frac{1 / 6}{10 / 6}$ |

Normalizing the weights:

| Event | Normalization |  | Probability |  |
| :---: | :---: | :---: | :---: | :---: |
| $M$ | $6 / 6(6 / 10)$ |  | 0.6 |  |
| $W$ | $3 / 6(6 / 10)$ | $=$ | 0.3 |  |
| $H$ | $1 / 6(6 / 10)$ | $=$ | 0.1 |  |
|  |  |  |  |  |

## Expected Value Models EMV \& EOL

Once a probability distribution has been assessed for each set of uncertain states of nature-and this can always be done, subjectively- it is straightforward to apply the next step called for by Laplace, namely, compute the expected value for each action alternative. Since there are two ways to look at the same problem (actual monetary values and opportunity losses), we can compute the expected values on either one of the payoff tables.

## Expected Monetary Value

It is possible to obtain probability estimates for each state of nature in decision-making situations. We use the expected monetary value criterion (used in Statistics) to identify the best decision alternative. The expected monetary value $\boldsymbol{E M V}$ is calculated by multiplying each decision outcome (payoff value) for each state of nature by the probability of its occurrence. Then the best decision is the one with the largest expected monetary value.

Using the original payoff matrix, the formula for expected monetary value (EMV) is:

$$
\mathrm{E}\left(A_{i}\right)=\Sigma_{j} p_{j}\left(R_{i j}\right)
$$

Thus, using the probability distribution derived previously:

Best decision is : Just Right plant

|  |  | ALTERNATIVES |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | PROBABILITIES | STATES <br> OF <br> NATURE | Large <br> plant | Just <br> Right <br> plant | Small <br> plant |


| 0.1 | High <br> demand | 15 | 9 | 3 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.6 | Medium <br> demand | 3 | 4 | 2 | 0 |
| 0.3 | Low <br> demand | -6 | -2 | 1 | 0 |
|  | EMV | 1.5 | $2.7^{*}$ | 1.8 | 0 |

$$
\max \mathrm{EMV}=\mathbf{E M V} *
$$

## Expected Opportunity Loss

An alternative to the above approach is the expected opportunity loss criterion (EOL). This utilizes regrets (opportunity losses) to minimize the expected regret. From the regret table with each state of nature assigned probability we calculate the expected opportunity loss (EOL) for each decision alternative

Using the opportunity loss matrix, the formula for expected opportunity loss (EOL) is:

$$
\mathrm{E}\left(A_{i}\right)=\Sigma_{j} p_{j}\left(O L_{i j}\right)
$$

Obviously, the same probability distribution applies (the states of nature are the same):

| PROBABILITIES | STATES <br> OF <br> NATURE | ALTERNATIVES |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Large plant | Just Right plant | Small plant | No plant |
| 0.1 | High demand | 0 | 6 | 12 | 15 |
| 0.6 | Medium demand | 1 | 0 | 2 | 4 |
| 0.3 | Low demand | 7 | 3 | 0 | 1 |
|  | EOL | 2.7 | 1.5* | 2.4 | 4.2 |

$$
\min \mathrm{EOL}=\mathbf{E O L} *
$$

The best decision results from minimizing the regret. In this case, the decision is a "Just Right Plant." The expected value and expected opportunity loss criteria result in the same decision. You may wonder why you need two separate approaches to reach the same conclusion. This will be discussed in the next section.

## The Relationship Between EMV and EOL

Note that both decision criteria (EMV and EOL) pointed to the same action alternative $J R$. Will this always be the case? Yes it will. To see why consider an uncannily accurate forecaster making this same exact decision a large number of times. This is hypothetical, of course. In reality, the decision situation is unique and will never be the same once the first decision is made, so repeatability is out of the question. But let's assume repeatability for the sake of discussion. Since the event market demand S is a random variable but our master forecaster never fails, she will predict $H 10 \%$ of the time, $M$ $60 \%$ of the time, and $W 30 \%$ of the time she makes the forecast. (Remember, the forecaster can predict but cannot control the outcome event. Consequently, her forecast record will mirror the probability distribution.) Now, with perfect forecasting she will never experience opportunity losses. The OL matrix shows this when an $O L_{i j}$ value is equal to zero. The corresponding $R_{i j}$ payoffs for those matrix cells are 15,4 , and 1 , respectively: the highest possible payoffs under the different market-demand conditions. Taking the expected value of these best-possible payoffs:

$$
\mathrm{E}\left(A^{*}\right)=0.1(15)+0.6(4)+0.3(1)=4.2
$$

where $A^{*}$ is the optimal action alternative for each state of nature. This means that the highest expected value possible for this problem (under conditions of infallible forecasts) is 4.2 . Note that this idealized expected payoff (or expected payoff given perfect forecasts) arises if and only if the expected opportunity loss is zero. Now, any expected opportunity loss that is incurred must come out of forgone expected payoffs, by definition. Since the maximum (idealized) expected payoff is fixed at 4.2 for this problem, and since the expected monetary value is what remains after an expected opportunity loss is deducted from the maximum expected payoff, the following equation holds:

$$
\mathrm{EMV}+\mathrm{EOL}=4.2 \quad \text { for this particular problem. }
$$

This is true for every action alternative. In general,

$$
\begin{aligned}
& \text { EMV }+ \text { EOL = Expected Payoff given Perfect Forecasts for all } \\
& A_{i} \text { in A. }
\end{aligned}
$$

Clearly, Max EMV can only be obtained with Min EOL. Thus, both criteria must point to the same $A_{i}$.

Critique of Expected Value Models
The fact that both EMV and EOL select the same action alternative $A_{i}$ is a welcome departure from our experience with the elementary models that did not use probability. The lack of consistency in recommending an action alternative exhibited by those models greatly reduces our confidence in them as reliable decision tools. EMV and EOL are certainly more robust in this sense. They also employ all of the available information about the problem, complying with a basic requirement of rationality. Another attractive aspect is that by making use of subjective probability, these models are able to incorporate the decision maker's personal impressions about future events. In other words, the models do not impose a "rigid theoretical solution" on the decision maker. Rather, the decision maker can adapt the model to conform to his/her judgment, intuition, experience and expectations.

In principle, expected value models work just fine. In practice, there is still one more point to examine: the subjectivity of utility. This will be done shortly.

## EVPI

## Expected Value of Perfect Information

In our last episode we left our intrepid and ever fearless managers in possession of a probability distribution they had derived subjectively. Now, even as Wolfgang Cactus expressed satisfaction with said distribution and accepted the results of the expected value models (EMV and EOL) as valid, Goldie Lockes had lingering doubts. "What if," she rhetorically asked, "the so-called said distribution, based on our limited knowledge about the problem situation (as must be the case because of bounded rationality), fails to reflect accurately the perilous nuances of risk that could be abridged with recourse to additional market information?" To which Cactus just stared dumbfounded.

Subjectively derived probability distributions are useful, yes, but there is no guarantee they are the best possible distributions if the subject (person) is not $100 \%$ informed about the problem situation. Most people on this planet are not $100 \%$ informed about anything. Consequently, it is generally possible to
obtain additional information about the problem that could be used to improve the accuracy of the subjective estimates. Obtaining additional problem information does not mean one should relinquish one's original assessment of the situation. After all, any other source of information is also subject to bounded rationality. Additional information should rationally be used to revise our prior estimates, not to supplant them (assuming, of course, the original estimates were not a haphazard guess).

Acquiring additional information involves work. Work implies expenses. When we buy something, are we willing to pay any price whatsoever for the purchase? If one is not a teenager buying recorded music, no. Everything has a price. The price reflects what the buyer is willing to pay in order to increase her/his satisfaction or well-being above and beyond the cost (setback) of obtaining the purchase. That is to say, one would be willing to acquire additional information if, and only if, the additional information translates into higher expected earnings. Otherwise, no dice.

Price is a function of quality. The higher the quality of a good, the more we'd be willing to pay for it. As regards information, higher quality means better accuracy. If the information is totally worthless, the price we'd be willing to pay is zero (no added benefit would accrue). If it's somewhat reliable, we'd be willing to pay something, though not much. If it's really good, we'd be willing to pay more. If it is perfect (infallible) information, how much would we be willing to pay? To be sure, there is a limit to the amount we'd be willing to pay: we would pay to the extent that the perfect information improves our expected earnings (assuming a rational decision maker). If obtaining the additional information reduces our net expected earnings, we'd rather do without the information.

So there is a maximum price we'd be willing to pay for perfect-absolutely infallible-information. Time to bring back our good friend, the uncannily accurate forecaster. She is so good she is actually referred to as a prophet, although an economic one at that. If we had access to such a prophetess, we would ask her what state of nature (market demand) was "destined" to occur and she would tell us. (Prophets are always nice guys and have to tell. Otherwise they wouldn't be called prophets.) The prophetess, keep in mind, only tells what is bound to occur; she does not alter "destiny."

If the prophet augurs that market demand for newfangled road runner traps is going to be high $(H)$, ACME's managers, conscious of her unerring predictions, would choose to build a large ( $L$ ) manufacturing plant (see payoff table):

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | STATES <br> OF <br> NATURE | Large <br> plant | Just <br> Right <br> plant | Small <br> plant | No <br> plant |
|  | High <br> demand | 15 | 9 | 3 | 0 |
| 0.1 | Medium <br> demand | 3 | 4 | 2 | 0 |
| 0.6 | Low <br> demand | -6 | -2 | 1 | 0 |
| 0.3 |  |  |  |  |  |

This can be done for every possible prophecy: $H, M, W$. Consequently, ACME's managers would know which decision is optimal given perfect information. This is shown in the decision tree below:


Decisions with Perfect Information

But ACME's managers do not know what the prophetess is going to foretell. (If they knew, they would not need to ask her.) So the prophecy itself is an uncertain state of affairs, as represented by the circle node in the decision tree above. However, by applying the probability distribution they already have (which represents the best information currently at their disposal), ACME's managers can compute the expected value of that decision tree, that is, the Expected Value given Perfect Information (EV|PI):

$$
\mathrm{EV} \mid \mathrm{PI}=\Sigma_{j} p_{j}\left(R_{i j}^{*}\right)
$$

where $R_{i j}{ }^{*}$ is the best payoff under state $S_{j}$. Thus EV $\mid$ PI $=4.2$. Which we already knew (perfect information is the same as perfect forecasts, but the former term is the standard nomenclature; thus, $\mathrm{EP}|\mathrm{PF}=\mathrm{EV}| \mathrm{PI})$. Does this mean that the prophetess's information is worth $\$ 4.2$ million to ACME's managers? No way! Cactus and Locke were able to "secure" an expected
monetary payoff of $\$ 2.7$ million on their own without the assistance of the prophetess (see EMV* on previous page). Hence, the prophetess should not be credited with the first $\$ 2.7$ million of expected payoffs. Only the expected amount above and beyond the initial (or a priori) expected payoff of $\$ 2.7$ million is due to her information. Therefore, the Expected Value of Perfect Information (EVPI) is:

$$
\mathrm{EVPI}=\mathrm{EV} \mid \mathrm{PI}-\mathrm{EMV} *
$$

which in ACME's case works out to $\$ 1.5$ million. Perfect information would increase ACME's expected payoff by $\$ 1.5$ million, so that is what the perfect information is worth (to ACME's managers). Note that 1.5 is the minimum expected opportunity loss (EOL*). Consequently, since EMV $+\mathrm{EOL}=\mathrm{EV} \mid \mathrm{PI}$ and EMV* $\rightarrow$ EOL*:
EVPI = EOL*

Or look at it this way: The EOL|PI is zero. EOL* is the minimum EOL without additional information. Thus, it is the additional perfect information that makes it possible to reduce the prior EOL* to zero. Hence, the value of this information is equal to its economic contribution: EOL* $-\mathrm{EOL} \mid \mathrm{PI}=$ EVPI.

Of course, prophets don't exist in economics, as we all know rather well, economics being the dismal science. But EVPI provides a criterion by which to judge ordinary mortal forecasters. If the cost of acquiring additional realworld information about ACME's market demand is greater than $\$ 1.5$ million, ACME should decline. It's not worth that much to ACME, irrespective of its degree of perfection. If real-world information were to cost less than $\$ 1.5$ million, should ACME's managers buy it? That depends on the quality of the information, remember. EVPI can be used to reject costly proposals but not to accept any forecasting offers because one needs to know the quality of the information one is acquiring. It may well be cheap, but it could be worthless. It is necessary to evaluate the quality of real-world (or imperfect) information.

## Terms

EVPI (Expected Value of Perfect Information) - the theoretical maximum worth to the decision maker of additional information about uncertain states of nature that is absolutely unerring.

EV|PI (Expected Value given Perfect Information) - the expected monetary value that would result if the decision maker had access to perfect information.

Now, try some problems:

1. XYZ company manufactures goods for a market in which the technology of the products is changing rapidly. The research and development department has produced a new product, which appears to have potential for commercial exploitation. A further Rs. 60,000 is required for development testing.

The company has 100 customer and each customer might purchase, at the most, one unit of the product. Market research suggests a selling price of Rs. 6,000 for each unit with total variable costs of manufacture and selling estimated at Rs. 2,000 for each unit.

As a result of previous experience of this type of market it has been possible to derive a probability distribution relating to the proportions of customers who will buy the product, as follows:

| Proportion of customers | Probability |
| :---: | :---: |
| 0.04 | 0.1 |
| 0.08 | 0.1 |
| 0.12 | 0.2 |
| 0.16 | 0.4 |
| 0.20 | 0.2 |

Determine the expected opportunity losses, given no other information than that stated above, and state whether, or not, the company should develop the product.
2. A businessman has two independent investments available to him but does not have the capital to undertake both of them simultaneously. He can choose to take A first and then stop, or if $A$ is successful then take $B$, or vice versa. The probability of success on $A$ is 0.7 while for B it is 0.4 . Both investments require an initial capital outlay of Rs. 20,000 , and both return nothing if the venture is unsuccessful. Investment A will return Rs. 30,000 (over cost) if it is successful, whereas successful completion of B will return Rs.50,000 (over cost). Using EMV as a decision criterion, decide the best strategy the businessman can adopt.
3. An oil company may bid for only one for the two contracts for oil drilling in two different areas. It is estimated that a profit of Rs. 30,000 would be realized from the first field and Rs. 40,000 from the second field. These profits amount have been determined ignoring the costs of bidding which amount to Rs. 2,500 for the first field and Rs. 5,000 for the second field which oil field the Co. Should bid for if the probability of getting contract for first field is 0.7 and that of second field is 0.6 ?

Ans. The company should bid for the second field.
4. Calculate the loss table from the following payoff table:

|  |  | Event |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Action | E1 | E2 | E3 | E4 |
| A $_{1}$ | 50 | 300 | -150 | 50 |
| A $_{2}$ | 400 | 0 | 100 | 0 |
| A $_{3}$ | -50 | 200 | 0 | 100 |
| A $_{4}$ | 0 | 300 | 300 | 0 |

Suppose that the probabilities of the events in this table are:

$$
P\left(\mathrm{E}_{1}\right)=0.15 ; \mathrm{P}\left(\mathrm{E}_{2}\right)=0.45 ; \mathrm{P}\left(\mathrm{E}_{3}\right)=0.25 ; \mathrm{P}\left(\mathrm{E}_{4}\right)=0.15
$$

Calculate the expected payoff and the expected loss of each action.
5. A company is trying to decide what size plant to build in a certain area. Three alternatives are being considered; plants with capacity of 20,000; 30,000 and 40,000 units respectively. Demand for the product is uncertain, but management has assigned the probabilities listed below to five levels of demand. The table below also shows the profit for each alternative and each possible level of demand (output may exceed rated capacity).

Payoff table showing profits (Cores of Rupees for various sizes of plants and levels of demand):

| Demand <br> Units | Probability | Profit (Rs. Crores) for different <br> Courses of Action-Build plant with capacity |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 20,000 units | 30,000 units | 40,000 units |
| 10,000 |  | -4.0 | -6.0 | -8.0 |
| 20,000 | 0.3 | 1.0 | 0.0 | -2.0 |
| 30,000 | 0.2 | 1.5 | 6.0 | 5.0 |
| 40,000 | 0.2 | 2.0 | 7.5 | 11.0 |
| 50,000 | 0.1 | 2.0 | 8.0 | 12.0 |

What size plant should be built?
6. A toy company is bringing out a new type of toy. The company is attempting to decide whether to bring out a full, partial, or minimal product line. The company has three level of product acceptance and has estimated their probability of occurrence. Management will make its decision on the basis of maximizing the expected profit from the year of production. The relevant data are show in the following table:

First-year Profit (Rs. ‘000)

|  |  | Product Line |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Product <br> Acceptance | Probability | Full | Partial | Minimum |
| Good | 0.2 | 80 | 70 | 50 |
| Fair | 0.4 | 50 | 45 | 40 |
| Poor | 0.4 | -25 | -10 | 0 |

(a) What is the optimum product line and its expected profit?
(b) Develop an opportunity loss table and calculate the EOL values. What is optimum value of EOL and the optimum course of action?
[Ans. (a) Partial; Rs. 28,000 (b) Rs. 8,000]
7. The Zeta Manufacturing Company Ltd. is proposing to introduce to the market a radio controlled toy car. It has three different possible models $\mathrm{X}, \mathrm{Y}$ and Z that vary in complexity but it has sufficient capacity to manufacture only one model. An analysis of the probable acceptance of the three models has been carried out and the resulting profit estimated:

| Model Acceptance | Probability | Annual Profits | (Rs. '000's) <br> X | Model Type |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Y | Z |  |
| Excellent | 0.3 | 120 | 100 | 60 |
| Moderate | 0.5 | 80 | 60 | 50 |
| Poor | 0.2 | -30 | -20 | 0 |

(i) Determine the model type that maximizes the expected profit. What is the expected profit?
(ii) Obtain an opportunity loss table and show that the difference between expected opportunity losses is the same as the difference between expected profits.
(iii) How much would it be worth to know the model acceptance level before making the decision on which model type to produce?

## CASE STUDY

## Ski Right

After retiring as a physician, Bob Guthrie became an avid downhill skier on the steep slopes of the Utah Rocky Mountains. As an amateur inventor, Bob was always looking for something new. With the recent deaths of several celebrity skiers, Bob knew he could use his creative mind to make skiing safer and his bank account larger. He knew that many deaths on the slopes were caused by head injuries. Although ski helmets have been on the market for some time, most skiers considered them boring and basically ugly. As a physician, Bob knew that some type of new ski helmet was the answer.

Bob's biggest challenge was to invent a helmet that was attractive, safe, and fun to wear. Multiple colors, using the latest fashion designs would be a must. After years of skiing, Bob knew that many skiers believed that how you looked on the slopes was more important than how you skied. His helmets would have to look good and fit in with current fashion trends. But attractive helmets were not enough. Bob had to make the helmets fun and useful. The name of the new ski helmet, Ski Right, was sure to be a winner. If Bob could come up with a good idea, he believed that there was a $20 \%$ chance that the market for the Ski Right Helmet would be excellent. The chance of a good market should be $40 \%$. Bob also knew that the market for his helmet could be only average ( $30 \%$ chance) or even poor ( $10 \%$ chance).

The idea of how to make ski helmets fun and useful came to Bob on a gondola ride to the top of a mountain. A busy executive on the gondola ride was on his cell phone trying to complete a complicated merger. When the executive got off of the gondola, he dropped the phone and it was crushed by the gondola mechanism. Bob decided that his new ski helmet would have a built-in cell phone and an AM/FM Stereo radio. All of the electronics could be operated by a control pad worn on a skier's arm or leg.

Bob decided to try a small pilot project for Ski Right. He enjoyed being retired and didn't want a failure to cause him to go back to work. After some research, Bob found Progressive Products (PP). The company was willing to be a partner in developing the Ski Right and sharing any profits. If the market were excellent, Bob would net $\$ 5,000$. With a good market, Bob would net $\$ 2,000$. An average market would result in a loss of $\$ 2,000$, and a poor market would mean Bob would be out $\$ 5,000$.

Another option for Bob was to have Leadville Barts (LB) make the helmet. The company had extensive experience in making bicycle helmets. Progressive would then take the helmets made by Leadville Barts and do the rest. Bob had a greater risk. He estimated that he could lose $\$ 10,000$ in a poor market or $\$ 4,000$ in an average market. A good market for Ski Right would result in a $\$ 6,000$ profit for Bob, while an excellent market would mean a $\$ 12,000$ profit.

A third option for Bob was to use TalRad TR, a radio company in Tallahassee, Florida. TalRad had extensive experience in making military radios. Leadville Barts
could make the helmets, and Progressive Products could do the rest. Again, Bob would be taking on greater risk. A poor market would mean a $\$ 15,000$ loss, while an average market would mean a $\$ 10,000$ loss. A good market would result in a net profit of $\$ 7,000$ for Bob. An excellent market would return \$13,000.

Bob could also have Celestial Cellular (CC) develop the cell phones. Thus, another option was to have Celestial make the phones and have Progressive do the rest of the production and distribution. Because the cell phone was the most expensive component of the helmet, Bob could lose $\$ 30,000$ in a poor market. He could lose $\$ 20,000$ in an average market. If the market were good or excellent, Bob would see a net profit of $\$ 10,000$ or $\$ 30,000$, respectively.

Bob's final option was to forget about Progressive Products entirely. He could use Leadville Barts to make the helmets, Celestial Cellular to make the phones, and TalRad to make the AM/FM stereo radios. Bob could then hire some friends to assemble everything and market the finished Ski Right helmets. With this final alternative, Bob could realize a net profit of $\$ 55,000$ in an excellent market. Even if the market were just good, Bob would net $\$ 20,000$. An average market, however, would mean a loss of $\$ 35,000$. If the market were poor, Bob would lose $\$ 60,000$.

## Discussion Questions

1. What do you recommend?
2. What is the opportunity loss for this problem?
3. Compute the expected value of perfect information.
4. Was Bob completely logical in how he approached this decision problem?

So, now let us summarize today's discussion:
Summary
We have discussed in details about Decision making under risk.

- Conceptual Approaches to Probability
- Expected Monetary Value
- Expected Opportunity Loss
- The Relationship Between EMV and EOL
- Expected Value of Perfect Information

