Unit 3
GAME THEORY

Lesson 30

Learning Objective:

- **Analyze two person non-zero sum games.**
- **Analyze games involving cooperation.**

Hello students,

The well-defined rational policy in neoclassical economics -- maximization of reward -- is extended to zero-sum games but not to the more realistic category of non-constant sum games.

"Solutions" to Non-constant Sum Games

The maximin strategy is a "rational" solution to all two-person zero sum games. However, it is not a solution for non-constant sum games. The difficulty is that there are a number of different solution concepts for non-constant sum games, and no one is clearly the "right" answer in every case. The different solution concepts may overlap, though.

We have already seen one possible solution concept for non-constant sum games: the dominant strategy equilibrium.

Let's take another look at the example of the two mineral water companies.

Two companies sell mineral water. Each company has a fixed cost of $5000 per period, regardless whether they sell anything or not. We will call the companies Perrier and Apollinaris, just to take two names at random.

The two companies are competing for the same market and each firm must choose a high price ($2 per bottle) or a low price ($1 per bottle). Here are the rules of the game:

1) At a price of $2, 5000 bottles can be sold for a total revenue of $10000.

2) At a price of $1, 10000 bottles can be sold for a total revenue of $10000.

3) If both companies charge the same price, they split the sales evenly between them.

4) If one company charges a higher price, the company with the lower price sells the whole amount and the company with the higher price sells nothing.

5) Payoffs are profits -- revenue minus the $5000 fixed cost.

Here is the payoff table for these two companies

Table 1

		Perrier	
		Price=$1	Price=$2
Apollinaris	Price=$1	0,0	5000, -5000
	Price=$2	-5000, 5000	0,0

We saw that the maximin solution was for each company to cut price to $1. That is also a dominant strategy equilibrium.

It's easy to check that: Apollinaris can reason that either Perrier cuts to $1 or not. If they do, Apollinaris is better off cutting to 1 to avoid a loss of $5000. But if Perrier doesn't cut, Apollinaris can earn a profit of 5000 by cutting. And Perrier can reason in the same way, so cutting is a dominant strategy for each competitor.

But this is, of course, a very simplified -- even unrealistic -- conception of price competition.

Let's look at a more complicated, perhaps more realistic pricing example:

Another Price Competition Example

Following a long tradition in economics, we will think of two companies selling "widgets" at a price of one, two, or three dollars per widget. The payoffs are profits -- after allowing for costs of all kinds -- and are shown in Table 2. The general idea behind the example is that the company that charges a lower price will get more customers and thus, within limits, more profits than the high-price competitor. (This example follows one by Warren Nutter).

Table 2

		Acme Widgets		
		p=1	p=2	p=3
Widgeon Widgets	p=1	0,0	50, -10	40,-20
	p=2	-10,50	20,20	90,10
	p=3	-20, 40	10,90	50,50

You can see that this is not a zero-sum game.

Profits may add up to 100, 20, 40, or zero, depending on the strategies that the two competitors choose. Thus, the maximin solution does not apply.

You can also see fairly easily that there is no dominant strategy equilibrium.

Widgeon company can reason as follows: if Acme were to choose a price of 3, then Widgeon's best price is 2, but otherwise Widgeon's best price is 1 -- neither is dominant.

Nash Equilibrium

You will need another, broader concept of equilibrium if you are to do anything with this game. The concept you need is called the Nash Equilibrium, after Nobel Laureate (in economics) and mathematician John Nash. Nash, a student of Tucker's, contributed several key concepts to game theory around 1950.

The Nash Equilibrium conception was one of these, and is probably the most widely used "solution concept" in game theory.

If there is a set of strategies with the property that no player can benefit by changing her strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute the **Nash Equilibrium**.

Let's apply that definition to the widget-selling game.

First, for example, you can see that the strategy pair p=3 for each player (bottom right) is not a Nash-equilibrium. From that pair, each competitor can benefit by cutting price, if the other player keeps her strategy unchanged. Or consider the bottom middle -- Widgeon charges $3 but Acme charges $2. From that pair, Widgeon benefits by cutting to $1. In this way, you can eliminate any strategy pair except the upper left, at which both competitors charge $1.

You will see that the Nash equilibrium in the widget-selling game is a low-price, zero-profit equilibrium. Many economists believe that result is descriptive of real, highly competitive markets -- although there is, of course, a great deal about this example that is still "unrealistic."

Let's go back and take a look at that dominant-strategy equilibrium in Table 1 of this lecture. You will see that it, too, is a Nash-Equilibrium. Check it out!

Also, look again at the dominant-strategy equilibrium in the Prisoners' Dilemma. It, too, is a Nash-Equilibrium. In fact, any dominant strategy equilibrium is also a Nash Equilibrium. The Nash equilibrium is an extension of the concepts of dominant strategy equilibrium and of the maximin solution for zero-sum games.

It would be nice to say that that answers all our questions.

But, of course, it does not…….

Here is just the first of the questions it does not answer:

- Could there be more than one Nash-Equilibrium in the same game?

- And what if there were more than one?

This leads us to a new concept

Games with Multiple Nash Equilibria

Here is another example to try the Nash Equilibrium approach on…..

Two radio stations (WIRD and KOOL) have to choose formats for their broadcasts. There are three possible formats: Country-Western (CW), Industrial Music (IM) or all-news (AN). The audiences for the three formats are 50%, 30%, and 20%, respectively. If they choose the same formats they will split the audience for that format equally, while if they choose different formats, each will get the total audience for that format. Audience shares are proportionate to payoffs. The payoffs (audience shares) are in Table 3.

Table 3

		KOOL		
		CW	IM	AN
WIRD	CW	25,25	50,30	50,20
	IM	30,50	15,15	30,20
	AN	20,50	20,30	10,10

You should be able to verify that this is a non-constant sum game, and that there are no dominant strategy equilibria.

If you find the Nash Equilibria by elimination, you will find that there are two of them -- the upper middle cell and the middle-left one, in both of which one station chooses CW and gets a 50 market share and the other chooses IM and gets 30. But it doesn't matter which station chooses which format.

It may seem that this makes little difference, since

- the total payoff is the same in both cases, namely 80
- both are efficient, in that there is no larger total payoff than 80

There are multiple Nash Equilibria in which neither of these things is so, as you will see in some later examples. But even when they are both true, the multiplication of equilibria creates a danger. The danger is that both stations will choose the more profitable CW format -- and split the market, getting only 25 each! Actually, there is an even worse danger that each station might assume that the other station will choose CW, and each choose IM, splitting that market and leaving each with a market share of just 15.

More generally, the problem for the players is to figure out which equilibrium will in fact occur. In other words, a game of this kind raises a "coordination problem:"

> How can the two stations coordinate their choices of strategies and avoid the danger of a mutually inferior outcome such as splitting the market?

Games that present coordination problems are sometimes called coordination games.

From a mathematical point of view, this multiplicity of equilibria is a problem. For a "solution" to a "problem," we want one answer, not a family of answers. And many economists would also regard it as a problem that has to be solved by some restriction of the assumptions that would rule out the multiple equilibria.

But, from a social scientific point of view, there is another interpretation. Many social scientists (myself included) believe that coordination problems are quite real and important aspects of human social life.

From this point of view, we might say that multiple Nash equilibria provide us with a possible "explanation" of coordination problems. That would be an important positive finding, not a problem!

Any bit of information that all participants in a coordination game would have, that could enable them all to focus on the same equilibrium.

In determining a national boundary, for example, the highest mountain between the two countries would be an obvious enough landmark that both might focus on setting the boundary there -- even if the mountain were not very high at all.

Another source of a hint that could solve a coordination game is social convention.

Here is a game in which social convention could be quite important. That game has a long name: "Which Side of the Road to Drive On?"

In Britain, you know, people drive on the left side of the road; in the US they drive on the right.

In abstract, how do we choose which side to drive on?

- There are two strategies: drive on the left side and drive on the right side.
- There are two possible outcomes: the two cars pass one another without incident or they crash.
- Arbitrarily assign a value of one each to passing without problems and of -10 each to a crash.

Here is the payoff table:

Table 4

		Mercedes	
		L	R
Buick	L	1,1	-10,-10
	R	-10,-10	1,1

Verify that LL and RR are both Nash equilibria. But, if we do not know which side to choose, there is some danger that we will choose LR or RL at random and crash.

How can we know which side to choose?

The answer is, of course, that for this coordination game we rely on social convention. Conversely, we know that in this game, social convention is very powerful and persistent, and no less so in the country where the solution is LL than in the country where it is RR.

Next, we move on to what is called as cooperative games.

Cooperative Games

Games in which the participants cannot make commitments to coordinate their strategies are "non-cooperative games." The solution to a "non-cooperative game" is a "non-cooperative solution."

In a non-cooperative game, the rational person's problem is to answer the question "What is the rational choice of a strategy when other players will try to choose their best responses to my strategy?"

Conversely, games in which the participants can make commitments to coordinate their strategies are "cooperative games," and the solution to a "cooperative game" is a "cooperative solution." In a cooperative game, the rational person's problem is to answer the question, "What strategy choice will lead to the best outcome for all of us in this game?"

If that seems excessively idealistic, you should keep in mind that cooperative games typically allow for "side payments," that is, bribes and quid pro quo arrangements so that every one is (might be?) better off.

Thus the rational person's problem in the cooperative game is actually a little more complicated than that. The rational person must ask not only "What strategy choice will lead to the best outcome for all of us in this game?" but also "How large a bribe may I reasonably expect for choosing it?"

A Basic Cooperative Game

Cooperative games are particularly important in economics. Here is an example that may illustrate the reason why.

- Suppose that Joey has a bicycle.
- Joey would rather have a game machine than a bicycle, and he could buy a game machine for $80, but Joey doesn't have any money. We express this by saying that Joey values his bicycle at $80.
- Mikey has $100 and no bicycle, and would rather have a bicycle than anything else he can buy for $100. We express this by saying that Mikey values a bicycle at $100.
- The strategies available to Joey and Mikey are to give or to keep. That is, Joey can give his bicycle to Mikey or keep it, and Mikey can give some of this money to Joey or keep it all.

- It is suggested that Mikey give Joey $90 and that Joey give Mikey the bicycle. This is what we call "exchange."

Here are the payoffs:

Table 5

		Joey	
		give	keep
Mikey	give	110, 90	10, 170
	keep	200, 0	100, 80

EXPLANATION:

- At the upper left, Mikey has a bicycle he values at $100, plus $10 extra, while Joey has a game machine he values at $80, plus an extra $10.

- At the lower left, Mikey has the bicycle he values at $100, plus $100 extra. At the upper left, Joey has a game machine and a bike, each of which he values at $80, plus $10 extra, and Mikey is left with only $10.

- At the lower right, they simply have what they begin with -- Mikey $100 and Joey a bike.

If we think of this as a non-cooperative game, it is much like a Prisoners' Dilemma. To keep is a dominant strategy and keep, keep is a dominant strategy equilibrium. However, give, give makes both better off. Being children, they may distrust one another and fail to make the exchange that will make them better off. But market societies have a range of institutions that allow adults to commit themselves to mutually beneficial transactions.

Thus, we would expect a cooperative solution, and we suspect that it would be the one in the upper left. But what cooperative "solution concept" may we use?

Pareto Optimum

We have observed that both participants in the bike-selling game are better off if they make the transaction. This is the basis for one solution concept in cooperative games.

First, we define a criterion to rank outcomes from the point of view of the group of players as a whole. We can say that one outcome is better than another (upper left better than lower right, e.g.) if at least one person is better off and no one is worse off. This is called the Pareto criterion, after the Italian economist and mechanical engineer, Vilfredo Pareto.

If an outcome (such as the upper left) cannot be improved upon, in that sense -- in other words, if no-one can be made better off without making somebody else worse off -- then we say that the outcome is Pareto Optimal, that is, Optimal (cannot be improved upon) in terms of the Pareto Criterion.

If there were a unique Pareto optimal outcome for a cooperative game, that would seem to be a good solution concept. The problem is that there isn't -- in general, there are infinitely many Pareto Optima for any fairly complicated economic "game."

In the bike-selling example, every cell in the table except the lower right is Pareto-optimal, and in fact any price between $80 and $100 would give yet another of the (infinitely many) Pareto-Optimal outcomes to this game. All the same, this was the solution criterion that von Neumann and Morgenstern used, and the set of all Pareto-Optimal outcomes is called the "solution set."

Alternative Solution Concepts

If we are to improve on this concept, we need to solve two problems.

- One is to narrow down the range of possible solutions to a particular price or, more generally, distribution of the benefits. This is called "the bargaining problem."
- Second, we still need to generalize cooperative games to more than two participants.

There are a number of concepts, including several with interesting results; but here attention will be limited to one. It is the Core, and it builds on the Pareto Optimal solution set, allowing these two problems to solve one another via "competition."

An Information Technology Example Revisited

When we looked at "Choosing an Information Technology," one of the two introductory examples, we came to the conclusion that it is more complex than the Prisoners' Dilemma in several ways. Unlike the Prisoners' Dilemma, it is a **cooperative game,** not a **non-cooperative game.** Now let's look at it from that point of view.

When the information system user and supplier get together and work out a deal for an information system, they are **forming a coalition** in game theory terms. (Here we have been influenced more by political science than economics, it seems!)

- The first decision will be whether to join the coalition or not. In this example, that's a pretty easy decision. Going it alone, neither the user nor the supplier can be sure of a payoff more than 0. By forming a coalition, both choosing the advanced system, they can get a total payoff of 40 between them.
- The next question is: how will they divide that 40 between them? How much will the user pay for the system?

We need a little more detail about this game before we can go on. The payoff table above was net of the payment. It was derived from the following gross payoffs:

Table 6

		User	
		Advanced	Proven
Supplier	Advanced	-50,90	0,0
	Proven	0,0	-30,40

The gross payoffs to the supplier are negative, because the production of the information system is a cost item to the supplier, and the benefits to the supplier are the payment they get from the user, minus that cost.

For Table 6, I assumed a payment of 70 for an advanced or 35 for a proven system. But those are not the only possibilities in either case. How much will be paid? Here are a couple of key points to move us toward an answer:

- The net benefits to the two participants cannot add up to more than 40, since that is the largest net benefit they can produce by working together.
- Since each participant can break even by going it alone, neither will accept a net less than zero.

Using that information, we get Figure A-1:

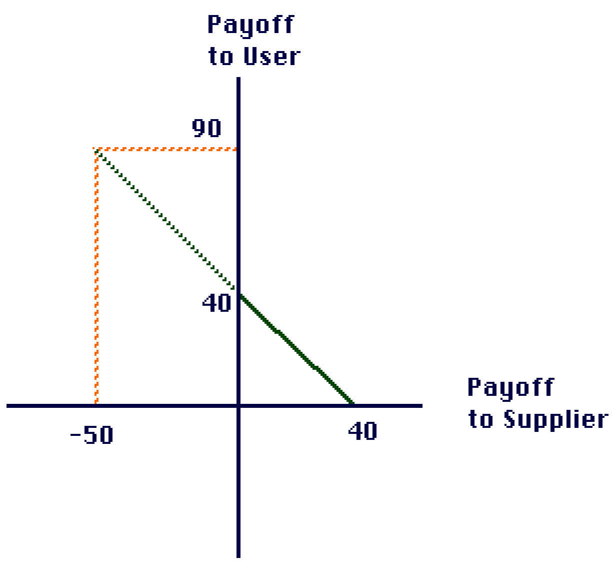

Figure A-1

The diagram shows the net payoff to the supplier on the horizontal axis and the net payoff to the user on the horizontal axis. Since the supplier will not agree to a payment that leaves her with a loss, only the solid green diagonal line -- corresponding to total payoffs of 40 to the two participants -- will be possible payoffs. But any point on that solid line will satisfy the two points above. In that sense, all the points on the line are possible "solutions" to the cooperative game, and von Neumann and Morgenstern called it the "solution set."

But this "solution set" covers a multitude of sins. How are we to narrow down the range of possible answers? There are several possibilities. The range of possible payments might be influenced, and narrowed, by:

- Competitive pressures from other potential suppliers and users,
- Perceived fairness,
- Bargaining.

There are game-theoretic approaches based on all these approaches, and on combinations of them. Unfortunately, this leads to several different concepts of "solutions" of cooperative games, and they may conflict. One of them -- the core, based on competitive pressures -- will be explored in these pages. We will have to leave the others for another time.

There is one more complication to consider, when we look at the longer run. What if the supplier does not continue to support the information system chosen? What if the supplier invests to support the system in the long run, and the user doesn't continue to use it? In other words, what if the commitments the participants make are limited by opportunism?

So, now let us summarize today's discussion:

Summary

We have discussed about:

- Several Examples
- Solution to Non-constant sum games
- Nash equilibrium
- Multiple Nash equilibrium
- Cooperative game
- Pareto Optimum.