Unit 3
GAME THEORY

Lesson 27

Learning Objective:
• **To learn to apply dominance in game theory.**
• **Generate solutions in functional areas of business and management.**

Hello students,
In our last lecture you learned to solve zero sum games having mixed strategies.

But...
Did you observe one thing that it was applicable to only 2 x 2 payoff matrices?

So let us implement it to other matrices using dominance and study the importance of **DOMINANCE**

In a game, sometimes a strategy available to a player might be found to be preferable to some other strategy / strategies. Such a strategy is said to dominate the other one(s). The rules of dominance are used to reduce the size of the payoff matrix. These rules help in deleting certain rows and/or columns of the payoff matrix, which are of lower priority to at least one of the remaining rows, and/or columns in terms of payoffs to both the players. Rows / columns once deleted will never be used for determining the optimal strategy for both the players.

This concept of domination is very usefully employed in simplifying the two – person zero sum games without saddle point. In general the following rules are used to reduce the size of payoff matrix.

The RULES (PRINCIPLES OF DOMINANCE) you will have to follow are:

Rule 1: If all the elements in a row (say i^{th} row) of a pay off matrix are less than or equal to the corresponding elements of the other row (say j^{th} row) then the player A will never choose the i^{th} strategy then we say i^{th} strategy is dominated by j^{th} strategy and will delete the i^{th} row.

Rule 2: If all the elements in a column (say r^{th} column) of a payoff matrix are greater than or equal to the corresponding elements of the other column (say s^{th} column) then the player B will never choose the r^{th} strategy or in the other words the r^{th} strategy is dominated by the s^{th} strategy and we delete r^{th} column .

Rule 3: A pure strategy may be dominated if it is inferior to average of two or more other pure strategies.

Now, consider some simple examples

Example 1

Given the payoff matrix for player A, obtain the optimum strategies for both the players and determine the value of the game.

Player B

Player A	6	-3	7
	-3	0	4

Solution

Player B

		B1	B2	B3
Player A	A1	6	-3	7
	A2	-3	0	4

When A chooses strategy A1 or A2, B will never go to strategy B3. Hence strategy

B3 is redundant.

		Player B		
		B1	B2	Row minima
Player A	A1	6	-3	-3
	A2	-3	0	-3
Column maxima		6	0	

Minimax (=0), maximin (= -3).Hence this is not a pure strategy with a saddle point.

Let the probability of mixed strategy of A for choosing Al and A2 strategies are p_1 and 1- p_1 respectively. We get

$$6\,p_1 - 3\,(1 - p_1) = -3\,p_1 + 0\,(1 - p_1) \quad \text{or} \quad p_1 = 1/4$$

Again, q_1 and 1 - q_1 being probabilities of strategy B, we get

$$6\,q_1 - 3\,(1 - q_1) = -3\,q_1 + 0\,(1 - q_1) \quad \text{or} \quad q_1 = 1/4$$

Hence optimum strategies for players A and B will be as follows:

$$S_A = \begin{bmatrix} A1 & A2 \\ 1/4 & 3/4 \end{bmatrix}$$

and

$$S_B = \begin{bmatrix} B1 & B2 & B3 \\ 1/4 & 3/4 & 0 \end{bmatrix}$$

Expected value of the game = $q_1(6p_1 - 3(1 - p_1)) + (1 - q_1)(3q_1 + 0(1 - q_1)) = \frac{3}{4}$

Example 2

In an election campaign, the strategies adopted by the ruling and opposition party alongwith pay-offs (ruling party's % share in votes polled) are given below:

Opposition Party's Strategies			
Ruling Party's Strategies	Campaign one day in each city	Campaign two days in large towns	Spend two days in large. rural sectors
Campaign one day in each city	55	40	35
Campaign two days in large towns	70	70	55
Spend two days in large rural sectors	75	55	65

Assume a zero sum game. Find optimum strategies for both parties and expected payoff to ruling party.

Solution. Let A1, A2 and A3 be the strategies of the ruling party and B1, B2 and B3 be those of the opposition. Then

Player B

		B1	B2	B3
Player A	A1	55	40	35
	A2	70	70	55
	A3	75	55	65

Here, one party knows his strategy as well as other party's strategy and one person's gain is another person's loss.

Now, with the given matrix:

		Player B			
		B1	B2	B3	Row minima
Player A	A1	55	40	35	35
	A2	70	70	55	55
	A3	75	55	65	65
Column maxima		75	70	65	

As maximin = 55 and minimax = 65 , there is no saddle point.

Row 1 is dominated by row 2 and column 1 is dominated by column 2 giving the reduced 2 x 2 matrix as :

	B2	B3
A2	70	55
A3	55	65

For ruling party: Let the ruling party select strategy A2 with a probability of p_1 and therefore opposition party selects strategy A3 with a probability of $(1 - p_1)$ Suppose the opposition selects strategy B2. Then the expected gain to ruling party for this game is given by :

$$70 p_1 + 55 (1 - p_1) = 15 p_1 + 55$$

On the other hand, if opposition party selects strategy B3, then ruling party's expected gain is :

$$55 p_1 + 65 (1- p_1) = -10 p_1 + 65$$

Now, in order for ruling party to be indifferent to which strategy, opposition party selects, the optimum plan for ruling party requires that its expected gain should be equal for each of opposition party's possible strategies. Thus equating two equations of expected gain, we get

$$15 p_1 + 55 = -10Pl + 65 \text{ or } p_1 = 2/5 \text{ and } 1 - p_1 = 3/5$$

Hence ruling party would select strategy A2 with probability of 0.4 and strategy A3 with probability of 0.6.

For opposition party. Let opposition party select strategy B2 and B3 with a probability of q_l and $(1 – q_1)$ respectively. The expected loss to opposition party when ruling party adopts strategy A2 and A3 respectively is :

$$70\ q_1 + 55(1 – q_1) = 15ql + 55 \text{ and } 55\ q_1 + 65\ (1 - q_l) = -10ql + 65$$

By equating expected losses of opposition party, regardless of what ruling party would choose, we get

$$15\ q_1 + 55 = -l0ql + 65 \text{ so that } q_1 = 2/5 \text{ and } (1 – q_1) = 3/5$$

Hence opposition party would choose strategy B2 and B3 with a probability of 0.4 and 0.6 respectively.

The value of the game is determined by substituting the value of p_l and q_1 in any of the expected values and is determined as 61, i. e.,

Expected gain to ruling party:

(i) 15 x 0.4 + 55 = 61

(ii)-10 x 0.4 + 65 = 61

Expected loss to opposition party:

(i) 15 x 0.4 + 55 = 61

(ii) -10 x 0.4 + 65 = 61

Example 3

Even though there are several manufacturers of scooters, two firms with branch names Janta and Praja, control their market in Western India. If both manufacturers make model changes of the same type for this market segment in the same year, their respective market shares remain constant. Likewise, if neither makes model changes, then also their market shares remain constant. The pay-off matrix in terms of increased/decreased percentage market share under different possible conditions is given below:

Praja

Janta	No change	Minor change	Major change
No change	0	-4	-10
Minor change	3	0	5
Major change	8	1	0

(i) Find the value of the game.

(ii) What change should Janta consider if this information is available only to itself? .

Solution.

(i) Clearly, the game has no saddle point. Making use of dominance principle, since the first row is dominated by the third row, we delete the first row. Similarly, first column is dominated by the second column and hence we delete the first column. The reduced pay-off matrix will be as follows:

	Praja	
Janta	Minor change	Major change
Minor change	0	5
Major change	1	0

As the reduced pay-off matrix does not possess any saddle point, the players will use mixed strategies. The optimum mixed strategy for player A is determined by :

$$p_1 = \frac{a_{22} - a_{21}}{a_{11}+a_{22} - (a_{12} + a_{21})} = \frac{0-1}{0+0-(5+1)} = \frac{1}{6}$$

$$p_2 = 1-p_1 = 5/6$$

$$q_1 = \frac{a_{22} - a_{12}}{a_{11}+a_{22} - (a_{12} + a_{21})} = \frac{0-5}{0+0-(5+1)} = \frac{5}{6}$$

$$q_2 = 1-q_1 = 1/6$$

The expected value of the game is given by

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{a_{11}+a_{22} - (a_{12} + a_{21})} = \frac{(0x0)-(1x5)}{0+0-(5+1)} = 5/6$$

Hence the optimal mixed strategies are

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 0 & 1/6 & 5/6 \end{bmatrix}$$

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 0 & 5/6 & 1/6 \end{bmatrix}$$

(ii) Janta may consider to have minor change with probability 5/6 and of major change with probability 1/6 .

Now,

Apply this method on some Unsolved Problems yourself

Q.1. Indicate the value of the game.

$$\begin{Vmatrix} 10 & 8 & 4 & 10 \\ 10 & 11 & 3 & 7 \\ 9 & 7 & 5 & 4 \end{Vmatrix}$$

Q.2. For the following 'two-person, zero-sum' game, find the optimal strategies for the two players and value of the game:

Player B

		B1	B2	B3
Player A	A1	9	9	3
	A2	6	-12	-11
	A3	8	16	10

Determine it using the principle of dominance.

Q.3. Assume that two firms are competing for market share for a particular product. Each firm is considering what promotional strategy to employ for the coming period. Assume that the following payoff matrix describes the increase in market share of Firm A and the decrease in market share for Firm B. Determine the optimal strategies for each firm.

Firm B

Firm A	Non Promotion	Moderate Promotion	Much Promotion
Non Promotion	5	0	-10
Moderate Promotion	10	6	12
Much Promotion	20	15	10

(i) Which firm would be the winner, in terms of market share?
(ii) Would the solution strategies necessarily maximize profits for either of the firms?
(iii) What might the two firms do to maximize their profits?

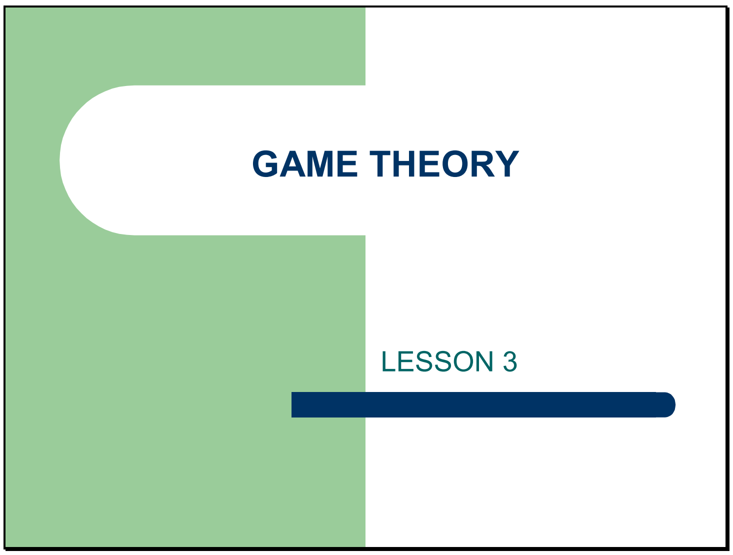
So, now let us summarize today's discussion:

Summary

We have discussed about:

- Importance of Dominance.
- Rules for Dominance.
- Applications of Dominance.

Slide 1



Slide 2

DOMINANCE

What is a Dominant Strategy?

A particular strategy that is found to be preferable over other strategies available to a player is called the “dominant strategy” for that player.

Slide 3

PRINCIPLES OF DOMINANCE

RULE 1

- If all the elements in a row (say i^{th} row) of a pay off matrix are less than or equal to the corresponding elements of the other row (say j^{th} row) then the player A will never choose the i^{th} strategy then we say i^{th} strategy is dominated by j^{th} strategy and will delete the i^{th} row.

Slide 4

PRINCIPLES OF DOMINANCE

RULE 2

- If all the elements in a column (say r^{th} column) of a payoff matrix are greater than or equal to the corresponding elements of the other column (say s^{th} column) then the player B will never choose the r^{th} strategy or in the other words the r^{th} strategy is dominated by the s^{th} strategy and we delete r^{th} column .

Slide 5

PRINCIPLES OF DOMINANCE

RULE 3

- A pure strategy may be dominated if it is inferior to average of two or more other pure strategies.