Unit 1

Lesson 20 :Solving Assignment problem

Learning objectives:

- ***Solve the assignment problem using Hungarian method.***
- ***Analyze special cases in assignment problems.***

Writing of an assignment problem as a Linear programming problem

Example 1. Three men are to to be given 3 jobs and it is assumed that a person is fully capable of doing a job independently. The following table gives an idea of that cost incurred to complete each job by each person:

Jobs → / Men ↓	J_1	J_2	J_3	Supply
M_1	20	28	21	1
M_2	15	35	17	1
M_3	8	32	20	1
Demand	1	1	1	

Formulate as a Linear programming problem.

Ans. The given problem can easily be formulated as a Linear Programming (transportation) model as under:

Minimize $Z = (20x_{11} + 28x_{12} + 21x_{13})$
$+ (15x_{21} + 35x_{22} + 17x_{23})$ (objective-function)
$+ (18x_{31} + 32x_{32} + 20x_{33})$

(it can also be written as: Minimise $Z = \sum_{i=1}^{3} \sum_{J=1}^{3} C_{iJ}\, x_{iJ}$

Subject to the following constraints:

(i)

$$x_{11} + x_{12} + x_{13} = 1$$

$$x_{21} + x_{22} + x_{23} = \sum_{i=1}^{3} 1 \qquad \text{or } x_{ij} = 1 \text{ where } i = 1, 2, 3$$

$$x_{31} + x_{32} + x_{33} = 1$$

(Since every person can be assigned only one job, therefore three constraint equation for three persons.)

(ii)

$$x_{11} + x_{21} + x_{31} = 1$$

$$x_{12} + x_{22} + x_{32} = \sum_{j=1}^{3} 1 \qquad \text{or } x_{ij} = 1 \text{ where J} = 1, 2, 3$$

$$x_{13} + x_{23} + x_{33} = 1$$

(Since each job can be assigned to only one person, therefore three equations for three different jobs)

(iii) $x_{ij} = \begin{cases} 1, \text{ if person I is assigned to job } J \\ 0, \text{ if person I is not assigned to job } J \end{cases}$

$\because a_i = b_J = 1$

$\Rightarrow$ the given problem is just a special case of the transportation problem.

Problems based on Hungarian Method

Example 2 :

A job has four men available for work on four separate jobs. Only one man can work on any one job. The cost of assigning each man to each job is given in the following table. The objective is to assign men to jobs such that the total cost of assignment is minimum.

	Jobs			
Persons	**1**	**2**	**3**	**4**
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D	25	23	24	24

Solution:

Step 1

Identify the minimum element in each row and subtract it from every element of that row.

Table

Jobs				
Persons	**1**	**2**	**3**	**4**
A	0	5	2	8
B	0	3	8	2
C	2	0	4	7
D	2	0	1	1

Step 2

Identify the minimum element in each column and subtract it from every element of that column.

Table

Jobs				
Persons	**1**	**2**	**3**	**4**
A	0	5	1	7
B	0	3	7	1
C	2	0	3	6
D	2	0	0	0

Step 3

Make the assignment for the reduced matrix obtain from **steps 1 and 2** in the following way:

a. Examine the rows successively until a row with exactly one unmarked zero is found. Enclose this zero in a box □as an assignment will be made there and cross (X) all other zeros appearing in the corresponding column as they will not be considered for future assignment. Proceed in this way until all the rows have been examined.

b. After examining all the rows completely, examine the columns successively until a column with exactly one unmarked zero is found. Make an assignment to this single zero by putting square □around it and cross out (X) all other assignments in that row, proceed in this manner until all columns have been examined.

c. Repeat the operations (a) and (b) successively until one of the following situations arises:

- All the zeros in rows/columns are either marked □or crossed (X) and there is exactly one assignment in each row and in each column. In such a case optimum assignment policy for the given problem is obtained.
- There may be some row (or column) without assignment, i.e., **<u>the total number of marked zeros is less than the order of the matrix. In such a case proceed to next step 4.</u>**

Table

Jobs				
Persons	1	2	3	4
A	[0]	5	1	7
B	0 (X)	3	7	1
C	2	[0]	3	6
D	2	0 (X)	[0]	0 (X)

Step 4

Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained from step 3 by adopting the following procedure:

i. Mark all the rows that do not have assignments.
ii. Mark all the columns (not already marked) which have zeros in the marked rows.
iii. Mark all the rows (not alreay marked) that have assignmets in marked columns.
iv. Repeat steps 4 (ii) and (iii) until no more rows or columns can be marked.
v. Draw straight lines through all unmarked rows and columns.

You can also draw the minimum number of lines by inspection

Table

	Jobs			
Persons	1	2	3	4
A	0	5	1	7
B	0	3	7	1
C	2	0	3	6
D	2	0	0	0

Step 5

Select the smallest element from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment.

Table

Jobs				
Persons	**1**	**2**	**3**	**4**
A	0	4	0	6
B	0	2	6	0
C	3	0	3	6
D	3	0	0	0

Go to ***step 3*** and repeat the procedure until you arrive at an optimum assignment.

Final Table

Jobs				
Persons	**1**	**2**	**3**	**4**
A	**[0]**	4	~~0~~	6
B	0	2	6	**[0]**
C	3	**[0]**	3	6
D	3	~~0~~	**[0]**	~~0~~

Since the number of assignments is equal to the number of rows (& columns), this is the optimal solution.

The total cost of assignment = A1 + B4 + C2 + D3

Substitute the values from original table:
20 + 17 + 24 + 17 = 78.

Example 3.

A departmental head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in there intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given the matrix below

Men				
Persons	**1**	**2**	**3**	**4**
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

Solution:

Step 1

Identify the minimum element in each row and subtract it from every element of that row, we get the reduced matrix

Table

Men				
Persons	**1**	**2**	**3**	**4**
A	7	15	6	0
B	0	15	1	13
C	23	4	3	0
D	9	16	14	0

Step 2

Identify the minimum element in each column and subtract it from every element of that column.

Table

Men				
Persons	**1**	**2**	**3**	**4**
A	7	11	5	0
B	0	11	0	13
C	23	0	2	0
D	9	12	13	0

Step 3

Make the assignment for the reduced matrix obtain from **steps 1 and 2** in the following way:

Now proceed as in the previous example

Optimal assignment is: A$\rightarrow$G, B $\rightarrow$ E, C $\rightarrow$F and D$\rightarrow$ H

The minimum total time for this assignment scheduled is 17 +13+19+10 or 59 man- hours.

Example 4: Time-matrix (Time in hrs.)

Men				
Persons	**1**	**2**	**3**	**4**
A	6	12	3	7
B	13	10	12	8
C	2	5	15	20
D	2	7	8	13

Solve this assignment problem. So as to minimize the time in hours.

Ans. Try yourself

Variation of Assignment Problem

Multiple Optimum Solutions

This situation of more than one optimal solutions the manager has a elasticity in decision making. Here the manager can choose any of the solutions by his will and experience.

Maximisation case in Assignment Problem

Some assignment problems entail maximizing the profit, effectiveness, or layoff of an assignment of persons to tasks or of jobs to machines. The Hungarian Method can also solve such problems, as it is easy to obtain an equivalent minimization problem by converting every number in the matrix to an opportunity loss. The conversion is accomplished by subtracting all the elements of the given effectiveness matrix from the highest element. It turns out that minimizing opportunity loss produces the same assignment solution as the original maximization problem.

Example 5:

Five different machines can do any of the five required jobs, with different profits resulting from each assignment as given below:

	Machines				
Jobs	**A**	**B**	**C**	**D**	**E**
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Find out the maximum profit possible through optimum assignment.

Solution:

Here, the highest element is **62**. So we subtract each value from 62.

Machines					
Jobs	**A**	**B**	**C**	**D**	**E**
1	32	25	22	34	22
2	22	38	35	41	26
3	22	30	29	32	27
4	37	24	22	26	26
5	33	0	21	28	23

Now use the **Hungarian Method** to solve the above problem.

The maximum profit through this assignment is 214.

Example 6. XYZ Ltd. employs 100 workers of which 5 are highly skilled workers that can be assigned to 5 technologically advanced machines. The profit generated by these highly skilled workers while working on different machines are as follows:

Machines → / **workers ↓**	**(Profit-matrix)**				
	III	**IV**	**V**	**VI**	**VII**
A	**40**	**40**	**35**	**25**	**50**
B	**42**	**30**	**16**	**25**	**27**
C	**50**	**48**	**40**	**60**	**50**
D	**20**	**19**	**20**	**18**	**25**
E	**58**	**60**	**59**	**55**	**53**

Solve the above assignment problem so as to maximize the profits of the company.

Unbalanced Assignment Problem

It is an assignment problem where the number of persons is not equal to the number of jobs.

If the number of persons is less than the number of jobs then we introduce one or more dummy persons (rows) with zero values to make the assignment problem balanced. Likewise, if the number of jobs is less than the number of persons then we introduce one or more dummy jobs (columns) with zero values to make the assignment problem balanced

Example 7 :

Jobs				
Persons	**1**	**2**	**3**	**4**
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24

Solution:

Since the number of persons is less than the number of jobs, we introduce a dummy person (D) with zero values. The revised assignment problem is given below:

Jobs				
Persons	**1**	**2**	**3**	**4**
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D (dummy)	0	0	0	0

Now use the **Hungarian Method** to solve the above problem.

Example8. In a typical assignment problem, four different machines are to be assigned to three different jobs with the restriction that exactly one machine is allowed for each job. The associated costs (in rupees ''000) are as follows:

Jobs			
Persons	**1**	**2**	**3**
A	60	80	50
B	50	30	60
C	70	90	40
D	80	50	70

Prohibited Assignment

Sometimes it may happen that a particular resource (say a man or machine) cannot be assigned to perform a particular activity. In such cases, the cost of performing that particular activity by a particular resource is considered to be very high (written as M or ∞) so as to prohibit the entry of this pair of resource-activity into the final solution.