Unit 1

Lesson 19: Assignment problem

Learning Objective :

- ***Recognize an Assignment problem.***
- ***Convert an assignment problem into a transportation problem.***
- ***State assignment problem in LP form.***

Introduction

In the world of trade Business Organisations are confronting the conflicting need for optimal utilization of their limited resources among competing activities. When the information available on resources and relationship between variables is known we can use LP very reliably. The course of action chosen will invariably lead to optimal or nearly optimal results. The problems which gained much importance under LP are :

Transportation problems (discussed in the previous chapter)

Assignment problems (covered under this chapter)

The assignment problem is a special case of transportation problem in which the objective is to assign a number of origins to the equal number of destinations at the minimum cost(or maximum profit). Assignment problem is one of the special cases of the transportation problem. It involves assignment of people to projects, jobs to machines, workers to jobs and teachers to classes etc., while minimizing the total assignment costs. One of the important characteristics of assignment problem is that only one job (or worker) is assigned to one machine (or project). Hence the number of sources are equal the number of destinations and each requirement and capacity value is exactly one unit.

Although assignment problem can be solved using either the techniques of Linear Programming or the transportation method, the assignment method is much faster and efficient. This method was developed by D. Konig, a Hungarian mathematician and is therefore known as the Hungarian method of assignment problem. In order to use this method, one needs to know only the cost of making all the possible assignments. Each assignment problem has a matrix (table) associated with it. Normally, the objects (or people) one wishes to assign are expressed in rows, whereas the columns represent the tasks (or things) assigned to them. The number in the table would then be the costs associated with each particular assignment. It may be noted that the assignment problem is a variation of transportation problem with two characteristics.(i)the cost matrix is a square matrix, and (ii)the optimum solution for the problem would be such that there would be only one assignment in a row or column of the cost matrix .

Mathematical Statement of Problem

An assignment problem is a special type of linear programming problem where the objective is to minimize the cost or time of completing a number of jobs by a number of persons. Furthermore, the structure of an assignment problem is identical to that of a transportation problem.

Application Areas of Assignment Problem.

Though assignment problem finds applicability in various diverse business situations, we discuss some of its main application areas:

(i) In assigning machines to factory orders.
(ii) In assigning sales/marketing people to sales territories.
(iii) In assigning contracts to bidders by systematic bid-evaluation.
(iv) In assigning teachers to classes.
(v) In assigning accountants to accounts of the clients.

In assigning police vehicles to patrolling areas.

		Jobs		
Persons	**j1**	**j2**	**------**	**jn**
I1	X11	X12	-----	X1n
I2	X21	X22	-----	X2n
---	-----	----	-----	-----
In	-----	----	-----	Xnn

Cij is the cost of performing jth job by ith worker. Xij is the number ith individual assigned to jth job.

Total cost = X11 * C11 + X12 * C12 + ----- + Xnn * Cnn.

Mathematically the assignment problem can be expressed as

The objective function is

Minimize C11X11 + C12X12 + ------- + CnnXnn.

This can also be written as:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} Xij = 1 \text{ for all i (resourse availability)}$$

$$\sum_{i=1}^{n} Xij = 1 \text{ for all i (activity requirement)}$$

and **Xij = 0 or 1, for all i to activity j.**

Solution Methods

The assignment problem can be solved by the following four methods :

- Enumeration method
- Simplex method
- Transportation method
- Hungarian method

1. Enumeration method

In this method, a list of all possible assignments among the given resources and activities is prepared. Then an assignment involving the minimum cost, time or distance or maximum profits is selected. If two or more assignments have the same minimum cost, time or distance, the problem has multiple optimal solutions. This method can be used only if the number of assignments is less. It becomes unsuitable for manual calculations if number of assignments is large

2. Simplex method

As discussed in chapter no. 2

3. **Transportation method**

As assignment is a special case of transportation problem it can also be solved using transportation model discussed in previous chapter. But the degeneracy problem of solution makes the transportation method computationally inefficient for solving the assignment problem.

4.Hungarian method

Algorithms for Solving

There are various ways to solve assignment problems. Certainly it can be formulated as a linear program (as we saw above), and the simplex method can be used to solve it. In addition, since it can be formulated as a network problem, the network simplex method may solve it quickly.

However, sometimes the simplex method is inefficient for assignment problems (particularly problems with a high degree of degeneracy). The Hungarian Algorithm developed by Kuhn has been used with a good deal of success on these problems and is summarized as follows.

Step 1. Determine the cost table from the given problem.

(i) If the no. of sources is equal to no. of destinations, go to step 3.
(ii) If the no. of sources is not equal to the no. of destination, go to step2.

Step 2. Add a dummy source or dummy destination, so that the cost table becomes a square matrix. The cost entries of the dummy source/destinations are always zero.

Step 3. Locate the smallest element in each row of the given cost matrix and then subtract the same from each element of the row.

Step 4. In the reduced matrix obtained in the step 3, locate the smallest element of each column and then subtract the same from each element of that column. Each column and row now have at least one zero.

Step 5. In the modified matrix obtained in the step 4, search for the optimal assignment as follows:

(a) Examine the rows successively until a row with a single zero is found. Enrectangle this row (□)and cross off (X) all other zeros in its column. Continue in this manner until all the rows have been taken care of.
(b) Repeat the procedure for each column of the reduced matrix.
(c) If a row and/or column has two or more zeros and one cannot be chosen by inspection then assign arbitrary any one of these zeros and cross off all other zeros of that row / column.
(d) Repeat (a) through (c) above successively until the chain of assigning (□) or cross (X) ends.

Step 6. If the number of assignment (□) is equal to n (the order of the cost matrix), an optimum solution is reached.

If the number of assignment is less than n(the order of the matrix), go to the next step.

Step7. Draw the **minimum number** of horizontal and/or vertical lines to cover all the zeros of the reduced matrix.

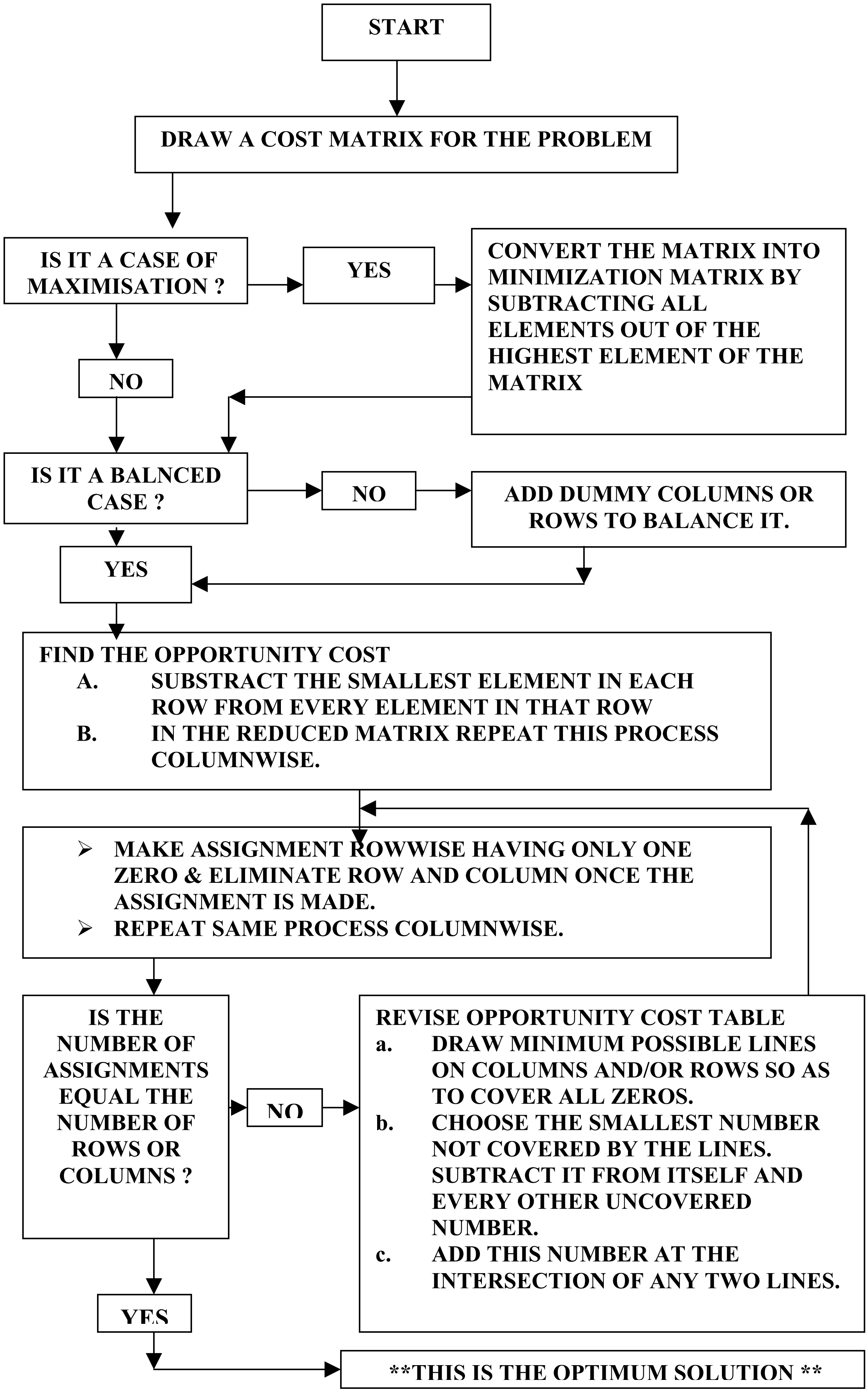
Step 8. Develop the new revised cost matrix as follows:

(a)Find the smallest element of the reduced matrix not covered by any of the lines.

(b)Subtract this element from all uncovered elements and add the same to all the elements laying at the intersection of any two lines.

Step 9. Go to step 6 and repeat the procedure until an optimum solution is attained.

See diagrammatic Representation of Hungarian Approach


START
DRAW A COST MATRIX FOR THE PROBLEM
IS IT A CASE OF MAXIMISATION ?
YES
CONVERT THE MATRIX INTO MINIMIZATION MATRIX BY SUBTRACTING ALL ELEMENTS OUT OF THE HIGHEST ELEMENT OF THE MATRIX
NO
IS IT A BALNCED CASE ?
NO
ADD DUMMY COLUMNS OR ROWS TO BALANCE IT.
YES
FIND THE OPPORTUNITY COST
A. SUBSTRACT THE SMALLEST ELEMENT IN EACH ROW FROM EVERY ELEMENT IN THAT ROW
B. IN THE REDUCED MATRIX REPEAT THIS PROCESS COLUMNWISE.
➢ MAKE ASSIGNMENT ROWWISE HAVING ONLY ONE ZERO & ELIMINATE ROW AND COLUMN ONCE THE ASSIGNMENT IS MADE.
➢ REPEAT SAME PROCESS COLUMNWISE.
IS THE NUMBER OF ASSIGNMENTS EQUAL THE NUMBER OF ROWS OR COLUMNS ?
NO
REVISE OPPORTUNITY COST TABLE
a. DRAW MINIMUM POSSIBLE LINES ON COLUMNS AND/OR ROWS SO AS TO COVER ALL ZEROS.
b. CHOOSE THE SMALLEST NUMBER NOT COVERED BY THE LINES. SUBTRACT IT FROM ITSELF AND EVERY OTHER UNCOVERED NUMBER.
c. ADD THIS NUMBER AT THE INTERSECTION OF ANY TWO LINES.
YES
**THIS IS THE OPTIMUM SOLUTION **