Unit 1

Lecture 18 **Special cases in Transportation Problems**

Learning Objectives:

Special cases in Transportation Problems
▪ ***Multiple Optimum Solution***
▪ ***Unbalanced Transportation Problem***
▪ ***Degeneracy in the Transportation Problem***
▪ ***Miximisation in a Transportation Problem***

Special cases

Some variations that often arise while solving the transportation problem could be as follows :

1.Multiple Optimum Solution
2.Unbalanced Transportation Problem
3.Degeneracy in the Transportation Problem

1.Multiple Optimum Solution

This problem occurs when there are more than one optimal solutions. This would be indicated when more than one unoccupied cell have zero value for the net cost change in the optimal solution. Thus a reallocation to cell having a net cost change equal to zero will have no effect on transportation cost. This reallocation will provide another solution with same transportation cost, but the route employed will be different from those for the original optimal solution. This is important because they provide management with added flexibility in decision making.

2.Unbalanced Transportation Problem

If the total supply is not equal to the total demand then the problem is known as unbalanced transportation problem. If the total supply is more than the total demand, we introduce an additional column, which will indicate the surplus supply with transportation cost zero. Similarly, if the total demand is more than the total supply, an additional row is introduced in the table, which represents unsatisfied demand with transportation cost zero.

Example1

Warehouses				
Plant	**W1**	**W2**	**W3**	**Supply**
A	28	17	26	500
B	19	12	16	300
Demand	250	250	500	

Solution:

The total demand is 1000, whereas the total supply is 800.
Total demand > total supply.
So, introduce an additional row with transportation cost zero indicating the unsatisfied demand.

Warehouses				
Plant	**W1**	**W2**	**W3**	**Supply**
A	28	17	26	500
B	19	12	16	300
Unsatisfied demand	0	0	0	200
Demand	250	250	500	1000

Now, solve the above problem with any one of the following methods:

- **North West Corner Rule**
- **Matrix Minimum Method**
- **Vogel Approximation Method**

Try it yourself.

Degeneracy in the Transportation Problem

If the basic feasible solution of a transportation problem with m origins and n destinations has fewer than m + n – 1 positive x_{ij} (occupied cells), the problem is said to be a degenerate transportation problem.

Degeneracy can occur at two stages:

1. At the initial solution
2. During the testing of the optimum solution

A degenerate basic feasible solution in a transportation problem exists if and only if some partial sum of availability's (row(s)) is equal to a partial sum of requirements (column(s)).

Example 2

Dealers					
Factory	**1**	**2**	**3**	**4**	**Supply**
A	2	2	2	4	1000
B	4	6	4	3	700
C	3	2	1	0	900
Requirement	900	800	500	400	

Solution:

Here, S1 = 1000, S2 = 700, S3 = 900
R1 = 900, R2 = 800, R3 = 500, R4 = 400

Since R3 + R4 = S3 so the given problem is a degeneracy problem.

Now we will solve the transportation problem by **Matrix Minimum Method**.

To resolve degeneracy, we make use of an artificial quantity(d). The quantity d is so small that it does not affect the supply and demand constraints.

Degeneracy can be avoided if we ensure that no partial sum of s_i (supply) and r_j (requirement) are the same. We set up a new problem where:

$s_i = s_i + d \quad i = 1, 2, ..., m$
$r_j = r_j$
$r_n = r_n + md$

Dealers					
Factory	**1**	**2**	**3**	**4**	**Supply**
A	**2** 900	**2** 100+d	2	4	1000 +d
B	4	**6** 700–d	**4** 2d	3	700 + d
C	3	2	**1** 500 –2d	**0** 400+3d	900 +d
Requirement	900	800	500	400 + 3d	

Substituting d = 0.

Dealers					
Factory	**1**	**2**	**3**	**4**	**Supply**
A	**2** 900	**2** 100	2	4	1000
B	4	**6** 700	**4** 0	3	700
C	3	2	**1** 500	**0** 400	900
Requirement	900	800	500	400 + 3d	

Initial basic feasible solution:

2 * 900 + 2 * 100 + 6 * 700 + 4 * 0 + 1 * 500 + 0 * 400 = 6700.

Now degeneracy has been removed.

To find the optimum solution, you can use any one of the following:

- **Stepping Stone Method.**
- **MODI Method.**

Miximisation in a Transportation Problem

Although transportation model is used to minimize transportation cost. However it can also be used to get a solution with an objective of maximizing the total value or returns.

Since the criterion of optimality is maximization, the converse of the rule for minimization will be used. The rule is : ***A solution is optimal if all – opportunity costs d_{ij} for the unoccupied cell are zero or negative.***

Let us take an example for this:

Example 3

A firm has three factories X, Y and Z. It supplies goods to four dealers spread all over the country. The production capacities of these factories are 200, 500 and 300 per month respectively.

Factory	**A**	**B**	**C**	**D**	**Capacity**
X	12	18	6	25	200
Y	8	7	10	18	500
Z	14	3	11	20	300
Demand	180	320	100	400	

Determine suitable allocation to maximize the total net return.

Solution:

Maximization transportation problem can be converted into minimization transportation problem by subtracting each transportation cost from maximum transportation cost.

Here, the maximum transportation cost is 25. So subtract each value from 25.

Factory	A	B	C	D	Capacity
X	13	7	19	0	200
Y	17	18	15	7	500
Z	11	22	14	5	300
Demand	180	320	100	400	

Now, solve the above problem by first finding initial solution any one of the following methods:

- **North West Corner Rule**
- **Matrix Minimum Method**
- **Vogel Approximation Method**

Then , testing the initial solution for optimality using either

- **Stepping stone method or**
- **MODI method**

Example 4.

A product is produced at three plants and shipped to three warehouses (the transportation costs per unit are shown in the following table.)

Warehouse				
Plant	W1	W2	W3	**Plant capacity**
P1	20	16	24	300
P2	10	10	8	500
P3	12	18	10	100
Warehouse demand	200	400	300	

a) Show a network representation of the problem.

b) Solve this model to determine the minimum cost solution.

c) c. Suppose that the entries in the table represent profit per unit produced at plant *I* and sold to warehouse *j.* How does the solution change from that in part (b) ?

Try it yourself.

Example 5.

Tri-county utilities, Inc.., supplies natural gas to customers in a three country area. The company purchases natural gas form two companies. Southern gas and Northwest Gas. Demand forecasts for the coming winter season are Hamilton county, 400 units; Butler county, 200 units: and Clermont county. 300 units. Contracts to provide the following quantities have been written. South Gas, 500 units; and Northwest Gas, 400 units, Distribution costs for the counties vary, depending upon the location of the suppliers. The distribution costs per unit (in thousands of dollars) are as follows:

To			
From	Hamilton	Butler	Clermont
Southern Gas	10	20	15
Northwest Gas	12	15	18

a. Develop a network representation of this problem.

b. Describe the distribution plan and show the total distribution cost.

c. Recent residential and industrial growth in Butter county has the potential for increasing demand by as much as 100 units. Which supplier should Tri- county contract with to supply the additional capacity?

Try it yourself.