

## Unit 1

### Lesson 17: Test for optimal solution to a transportation

#### Problem

#### Learning Objective:

**Test for Optimality**

- ***Modified Distribution Method (MODI)***

#### Modified Distribution Method (MODI)

It is a method for computing optimum solution of a transportation problem.

#### STEPS

##### Step 1

Determine an initial basic feasible solution using any one of the three methods given below:

- **North West Corner Rule**
- **Matrix Minimum Method**
- **Vogel Approximation Method**

##### Step 2

Determine the values of dual variables,  $u_i$  and  $v_j$ , using  $u_i + v_j = c_{ij}$

##### Step 3

Compute the opportunity cost using  $c_{ij} - (u_i + v_j)$ .

##### Step 4

Check the sign of each opportunity cost. If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimum solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimum solution and further savings in transportation cost are possible.

### Step 5

Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.

### Step 6

Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

### Step 7

Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.

### Step 8

Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. In this way an unoccupied cell becomes an occupied cell.

### Step 9

Repeat the whole procedure until an optimum solution is obtained.

### Example 1

A company is spending Rs. 1000 on transportation of its units from these plants to four distribution centers. The supply and requirement of units, with unity cost of transportation are given as:

Distribution centres						
		D1	D2	D3	D4	Supply
Plants	P1	19	30	50	12	7
	P2	70	30	40	60	10

	<b>P3</b>	40	10	60	20	18
<b>Requirement</b>		5	8	7	15	

Determine the optimum solution of the above problem.

**Solution:**

Now, solve the above problem by **Matrix Minimum Method.**

Distribution centres						
		D1	D2	D3	D4	Supply
Plants	P1	19	30	50	12 <sup>7</sup>	7
	P2	70 <sup>3</sup>	30	40 <sup>7</sup>	60	10
	P3	40 <sup>2</sup>	10 <sup>8</sup>	60	20 <sup>8</sup>	18
Requirement		5	8	7	15	

The solution is basic feasible as there are  $m + n - 1$ , i.e.,  $4 + 3 - 1 = 6$  allocations in independent solutions.

**Initial basic feasible solution:**

$$12 * 7 + 70 * 3 + 40 * 7 + 40 * 2 + 10 * 8 + 20 * 8 = \text{Rs. } 894.$$

**Calculating  $u_i$  and  $v_j$  using  $u_i + v_j = c_{ij}$**

Substituting  $u_1 = 0$ , we get

$$u_1 + v_4 = c_{14} \Rightarrow 0 + v_4 = 12 \text{ or } v_4 = 12$$

$$u_2 + v_4 = c_{24} \Rightarrow u_2 + 12 = 60 \text{ or } u_2 = 38$$

$$u_2 + v_3 = c_{23} \Rightarrow 38 + v_3 = 40 \text{ or } v_3 = 2$$

$$u_3 + v_4 = c_{34} \Rightarrow u_3 + 12 = 20 \text{ or } u_3 = 8$$

$$u_3 + v_2 = c_{32} \Rightarrow 8 + v_2 = 10 \text{ or } v_2 = 2$$

$$u_3 + v_1 = c_{31} \Rightarrow 8 + v_1 = 40 \text{ or } v_1 = 32$$

Distribution centres							
		D1	D2	D3	D4	Supply	$u_i$
Plants	P1	19	30	50	12 <sup>7</sup>	7	0
	P2	70 <sup>3</sup>	30	40 <sup>7</sup>	60	10	38
	P3	40 <sup>2</sup>	10 <sup>8</sup>	60	20 <sup>8</sup>	18	8
Requirement		5	8	7	15		
$v_j$		32	2	2	12		

Calculating opportunity cost using  $c_{ij} - (u_i + v_j)$

Unoccupied cells	Opportunity cost
(P <sub>1</sub> , D <sub>1</sub> )	$C_{11} - (u_1 + v_1) = 19 - (0 + 32) = -13$
(P <sub>1</sub> , D <sub>2</sub> )	$C_{12} - (u_1 + v_2) = 30 - (0 + 2) = 28$
(P <sub>1</sub> , D <sub>3</sub> )	$C_{13} - (u_1 + v_3) = 50 - (0 + 2) = 48$
(P <sub>2</sub> , D <sub>2</sub> )	$C_{22} - (u_2 + v_2) = 30 - (38 + 2) = -10$
(P <sub>2</sub> , D <sub>4</sub> )	$C_{14} - (u_2 + v_4) = 60 - (38 + 12) = 10$
(P <sub>3</sub> , D <sub>3</sub> )	$C_{33} - (u_3 + v_3) = 60 - (8 + 2) = 50$

Distribution centres							
		D1	D2	D3	D4	Supply	$u_i$
Plants	P1	-13 19	28 30	48 50	12 <sup>7</sup>	7	0
	P2	70 <sup>3</sup>	-10 30	40 <sup>7</sup>	10 60	10	38
	P3	40 <sup>2</sup>	10 <sup>8</sup>	50 60	20 <sup>8</sup>	18	8

<b>Requirement</b>		5	8	7	15		
$v_j$		32	2	2	12		

Distribution centres							
		D1	D2	D3	D4	Supply	$u_i$
<b>Plants</b>	P1	-13 19 <sub>+</sub>	28 30	48 50	12 7 <sub>-</sub>	7	0
	P2	70 3 <sub>+</sub>	-10 30	40 7 <sub>+</sub>	10 60	10	38
	P3	40 2 <sub>-</sub>	10 8 <sub>+</sub>	50 60	20 8 <sub>+</sub>	18	8
<b>Requirement</b>		5	8	7	15		
$v_j$		32	2	2	12		

Choose the smallest negative value from opportunity cost (i.e., -13). Now draw a closed path from P1D1 .

Choose the smallest value with a negative position on the closed path(i.e., 2), it indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. In this way an unoccupied cell becomes an occupied cell.

Now again calculate the values for  $u_i$  and  $v_j$ .

Distribution centres							
		D1	D2	D3	D4	Supply	$u_i$
<b>Plants</b>	P1	19 2 <sub>+</sub>	28 30	61 50	12 5 <sub>+</sub>	7	0
	P2	70 3 <sub>+</sub>	-23 30	40 7 <sub>+</sub>	-3 60	10	51
	P3	13 40	10 8 <sub>+</sub>	63 60	20 10 <sub>+</sub>	18	8
<b>Requirement</b>		5	8	7	15		
$v_j$		19	2	-11	12		

Now again we draw a closed path.

Distribution centres							
		D1	D2	D3	D4	Supply	$u_i$
Plants	P1	19 <sup>2+</sup>	28 30	61 50	12 <sup>5-</sup>	7	0
	P2	70 <sup>3-</sup>	-23 30 <sup>+</sup>	40 <sup>7</sup>	-3 60	10	51
	P3	13 40	10 <sup>8-</sup>	63 60	20 <sup>10+</sup>	18	8
Requirement		5	8	7	15		
$v_j$		19	2	-11	12		

Distribution centres							
		D1	D2	D3	D4	Supply	$u_i$
Plants	P1	19 <sup>5</sup>	8 30	28 50	12 <sup>2</sup>	7	0
	P2	33 70	30 <sup>3</sup>	40 <sup>7</sup>	30 60	10	18
	P3	23 40	10 <sup>5</sup>	40 60	20 <sup>13</sup>	18	-2
Requirement		5	8	7	15		
$v_j$		19	12	22	12		

Since all the current opportunity costs are non-negative, this is the optimum solution.

So the minimum transportation cost is:

$$19 * 5 + 12 * 2 + 30 * 3 + 40 * 7 + 10 * 5 + 20 * 13 = \text{Rs. } 799$$

### Example 2

Given below are the costs of shipping a product from various warehouses to different stores:

Warehouses	Stores (costs in rupees)				Supply
	1	2	3	4	
A	7	3	5	5	34
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
Demand	21	25	17	17	80

Generate an initial feasible solution & check it for optimality.

Try yourself

### Example 3

Alpha Ltd. has 3 factories (A, B, C) & warehouses (P, Q, R, S). It supplies the finished goods from each of its three factories to the four warehouses, the relevant shipping cost of which are as follows:

Factory → ↓ Warehouse	A	B	C
P	4	3	7
Q	5	8	4
R	2	4	7
S	5	8	4

Assuming the monthly production capacity of A, B & C to be 120, 80 & 200 tons respectively, determine the optimum transportation schedule so as to minimize the total transportation costs by using Vogel's method.

(The monthly requirements for the warehouses are as follows:

Warehouse	Monthly requirement (tons)
P	60
Q	50
R	140
S	50
Total 300 (tons)	

Try yourself

**Example 4.**

A company has three factories and five warehouses, where the finished goods are shipped from the factories. The following relevant details are provided to you: -

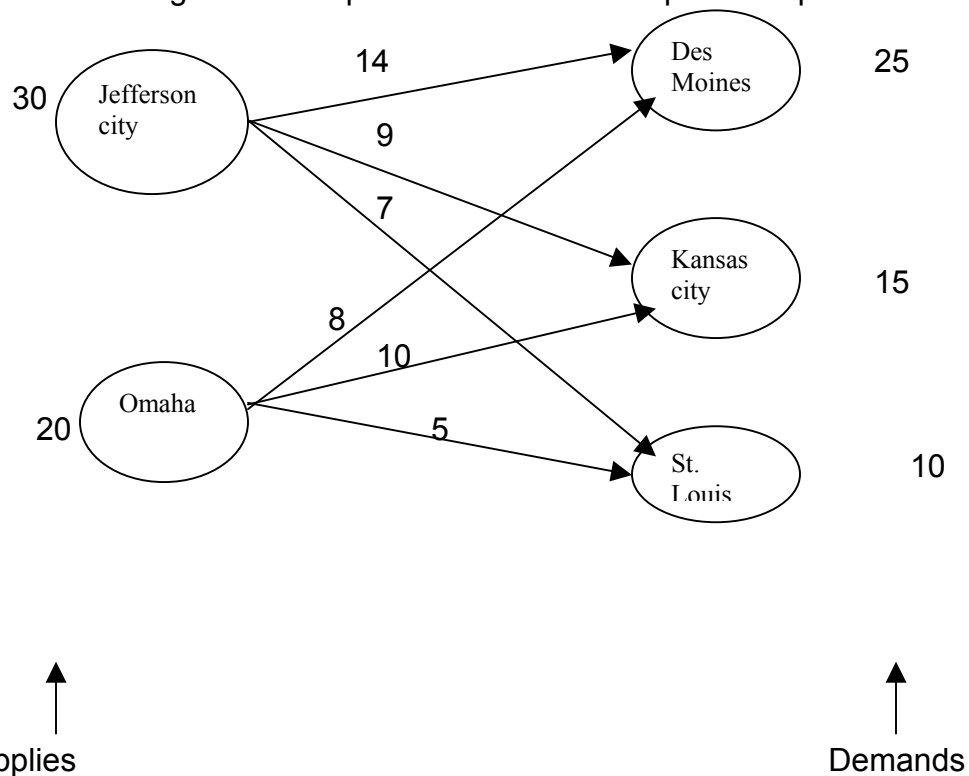
Warehouses→		(Costs in Rs.)					
Factory ↓							
		1	2	3	4	5	Availability
1		3	5	8	9	11	20
2		5	4	10	7	10	40
3		2	5	8	7	5	30
	<b>Requirement</b>	<b>10</b>	<b>15</b>	<b>25</b>	<b>30</b>	<b>40</b>	<b>120/90</b>

Solve the above transportation problem.

**Try yourself**

**Example 5.**

Consider the following network representation of a transportation problem.



The supply, demand and transportation costs per units are shown on the network.

- a. Develop a Transportation model for this problem.



b. Solve the Transportation model to determine the optimal solution.

**Try yourself**

