Unit 1

Lesson 16: Test for Optimal solution to a Transportation Problem

Learning Objective:

Test for Optimality

- ***Stepping Stone Method***

Before learning the methods to find the optimal solution try and practice few more questions to find the initial solution of the transportation problem.

Question1

Find the initial basic feasible solution to the following transportation problem using

1)North west corner rule (NWCR)
2)Matrix Minima Method (MMM)
3)Vogel's Approximation Method (VAM)

Retail shops					
Factories	1	2	3	4	Supply
1	11	13	17	14	250
2	16	18	14	10	300
3	21	24	13	10	400
Demand	200	275	275	250	

Example 2

Find the initial basic feasible solution to the following transportation problem using

1)North west corner rule (NWCR)
2)Matrix Minima Method (MMM)
3)Vogel's Approximation Method (VAM)

Destination					
Origin	**1**	**2**	**3**	**4**	**Supply**
1	1	2	3	4	6
2	4	3	2	0	8
3	0	2	2	1	10
Demand	4	6	8	6	24

Test for Optimality

Once the initial feasible solution is reached, the next step is to check the optimality. An optimal solution is one where there is no other set of transportation routes (allocations) that will further reduce the total transportation cost. Thus, we'll have to evaluate each unoccupied cell (represents unused routes) in the transportation table in terms of an opportunity of reducing total transportation cost.

1.Stepping Stone Method

It is a method for computing optimum solution of a transportation problem.

Steps

Step 1

Determine an initial basic feasible solution using any one of the following:

- **North West Corner Rule**
- **Matrix Minimum Method**
- **Vogel Approximation Method**

Step 2

Make sure that the number of occupied cells is exactly equal to m+n-1, where m is the number of rows and n is the number of columns.

Step 3

Select an unoccupied cell.

Step 4

Beginning at this cell, trace a closed path using the most direct route through at least three occupied cells used in a solution and then back to the original occupied cell and moving with only horizontal and vertical moves. The cells at the turning points are called "Stepping Stones" on the path.

Step 5

Assign plus (+) and minus (-) signs alternatively on each corner cell of the closed path just traced, starting with the plus sign at unoccupied cell to be evaluated.

Step 6

Compute the net change in the cost along the closed path by adding together the unit cost figures found in each cell containing a plus sign and then subtracting the unit costs in each square containing the minus sign.

Step 7

Check the sign of each of the net changes. If all the net changes computed are greater than or equal to zero, an optimum solution has been reached. If not, it is possible to improve the current solution and decrease the total transportation cost.

Step 8

Select the unoccupied cell having the most negative net cost change and determine the maximum number of units that can be assigned to a cell marked with a minus sign on the closed path corresponding to this cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign. Subtract this number from cells on the closed path marked with a minus sign.

Step 9

Repeat the procedure until you get an optimum solution.

Example 3

Consider the following transportation problem (cost in rupees). Find the optimum solution

Depot					
Factory	**D**	**E**	**F**	**G**	**Capacity**
A	4	6	8	6	700
B	3	5	2	5	400
C	3	9	6	5	600
Requirement	400	450	350	500	1700

Solution:

First, find out an initial basic feasible solution by **Matrix Minimum Method**

Depot					
Factory	**D**	**E**	**F**	**G**	**Capacity**
A	4	**6** 450	8	**6** 250	700
B	**3** 50	5	**2** 350	5	400
C	**3** 350	9	6	**5** 250	600
Requirement	400	450	350	500	1700

Here, m + n - 1 = 6. So the solution is not degenerate.

The cell AD (4) is empty so allocate one unit to it. Now draw a closed path from AD.

Depot					
Factory	**D**	**E**	**F**	**G**	**Capacity**
A	**4** $^{+1}$	**6** ^{450}	8	**6** ^{249}	700
B	**3** ^{50}	5	**2** ^{350}	5	400
C	**3** ^{349}	9	6	**5** ^{251}	600
Requirement	400	450	350	500	1700

Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

The increase in the transportation cost per unit quantity of reallocation is
+4 – 6 + 5 – 3 = 0.

This indicates that every unit allocated to route AD will neither increase nor decrease the transportation cost. Thus, such a reallocation is unnecessary.

Choose another unoccupied cell. The cell BE is empty so allocate one unit to it. Now draw a closed path from BE

Depot					
Factory	**D**	**E**	**F**	**G**	**Capacity**
A	**4**	**6** ^{449}	8	**6** ^{251}	700
B	**3** ^{49}	**5** $^{+1}$	**2** ^{350}	5	400
C	**3** ^{351}	9	6	**5** ^{249}	600
Requirement	400	450	350	500	1700

The increase in the transportation cost per unit quantity of reallocation is
+5 – 6 + 6 – 5 + 3 – 3 = 0

This indicates that every unit allocated to route BE will neither increase nor decrease the transportation cost. Thus, such a reallocation is unnecessary.

The allocations for other unoccupied cells are:

Unoccupied cells	Increase in cost per unit of reallocation	Remarks
CE	+9 – 6 + 6 – 5 = 4	Cost Increases
CF	+6 – 3 + 3 – 2 = 4	Cost Increases
AF	+8 – 6 +5 – 3 + 3 – 2 = 5	Cost Increases
BG	+5 – 5 + 3 – 3 = 0	Neither increase nor decrease

Since all the values of unoccupied cells are greater than or equal to zero, the solution obtained is optimum.

Minimum transportation cost is:
6 * 450 + 6 * 250 + 3 * 250 + 2 * 250 +3 *350 +5 * 250 = Rs. 7350

Example 4

A company has factories at A,B and C which supply warehouse at D,E and F. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly Warehouse requirement are 180, 120 and 150 units respt.. Unit shipping costs (in rupees)are as follows.

Factory	D	E	F	Capacity
A	16	20	12	200
B	14	8	18	160
C	26	24	16	90
Requirement	180	120	150	450

Determine the optimum distribution for this company to minimize shipping cost.

Solution:

First, find out an initial basic feasible solution by Vogel's approximation Method

Factory	D	E	F	Capacity
A	**16** 140	**20**	**12** 60	200
B	**14** 40	**8** 120	**18**	160
C	26	24	**16** 90	90
Requirement	180	120	150	450

Here, m + n - 1 = 5 So the solution is not degenerate.

The cell AE (20) is empty so allocate one unit to it. Now draw a closed path from AD.

Factory	D	E	F	Capacity
A	**16** 139	**20** +1	**12** 60	200
B	**14** 41	**8** 119	**18**	160
C	26	24	**16** 90	90
Requirement	180	120	150	450

The increase in the transportation cost per unit quantity of reallocation is
+20 – 8 + 14 – 16 = 10.

This indicates that every unit allocated to route AD will increase the transportation cost by Rs.10. Thus, such a reallocation is not included.

Choose another unoccupied cell. The cell BF is empty so allocate one unit to it. Now draw a closed path from BE

Factory	D	E	F	Capacity
A	**16** 141	**20**	**12** 59	200
B	**14** 39	**8** 120	**18** +1	160
C	26	24	**16** 90	90
Requirement	180	120	150	450

The opportunity cost in the transportation cost per unit quantity of reallocation is
+18 – 12 + 16 – 14 = 8

This indicates that every unit allocated to route BE will increase the transportation cost by Rs. 8. Thus, such a reallocation is not included.

The allocations for other unoccupied cells are:

Unoccupied cells	Increase in cost per unit of reallocation	Remarks
CD	+26 – 16 + 12 – 16 = 6	Cost Increases
CE	+24 – 16 + 12 – 16 +14 – 8= 10	Cost Increases

Since all the values of unoccupied cells are greater than zero, the solution obtained is optimum.

Minimum transportation cost is:
16 * 140 + 12 * 60 + 14 * 40 + 8 * 120 +16 *90 = Rs. 5,920

Example 5

Maruti Udyog Limited (MUL) produces & sells cars. It has two warehouses (A, B) and three whole sellers (1,2,3). The status at present in the warehouses is as follows:

Warehouse	Cars
A	40
B	60
	Total 100

The wholesalers have placed the following order from the month:

Whole seller	Demand (Cars)
1	20
2	30
3	40

The transportation charges (in shipping from warehouse to the whole seller) are given below:

Whole sellers / Warehouse	(Costs in Rs. '000)		
A	2	4	3
B	5	2	4

Your are required to determine the optimal number of cars to be shipped for each warehouse to each whole seller so as to minimize the total transportation costs.

Example 6.

Good Manufactures Limited has 3 warehouses (A, B, C) AND FOUR STORES (W, X, Y, Z). For a particular product, there is a surplus of 150 units at all the warehouse taken together, as given below:

Warehouse	**Units of product**
A	**50**
B	**60**
C	**40**
Total	**150**

The transportation costs of shipping one unit of the product from warehouses to stories is given below:

Stores / Warehouse	Costs (in Rs)			
	W	X	Y	Z
A	50	150	70	60
B	80	70	90	10
C	15	87	79	81

The monthly requirement of the stores is as follows:

Store	Requirement (of product)
W	20
X	70
Y	50

Z	10

You are required to obtain the optimal solution to the given transportation problem.