

## Unit 1

### Lesson 15: Methods of finding initial solution for a transportation problem.

#### Learning objective

Various Methods for finding initial solution to a transportation problem

1. North – west corner method
2. Minimum Matrix Method (MMM)
3. Vogel's Approximation Method (VAM)

#### Methods of finding initial solution

There are several methods of finding initial basis feasible solution. Here we shall discuss only three of them.

##### 1. North-West corner method (NWCM)

The North West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North – West corner (i.e., top left corner).

##### Steps

1. Select the north west (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirements, i.e.,  $\min(s_1, d_1)$ .
2. Adjust the supply and demand numbers in the respective rows and columns allocation.
3. If the supply for the first row is exhausted then move down to the first cell in the second row.
4. If the demand for the first cell is satisfied then move horizontally to the next cell in the second column.
5. If for any cell supply equals demand then the next allocation can be made in cell either in the next row or column.
6. Continue the procedure until the total available quantity is fully allocated to the cells as required.

**Example 2**

Retail shops					
Factories	1	2	3	4	Supply
1	3	5	7	6	50
2	2	5	8	2	75
3	3	6	9	2	25
Demand	20	20	50	60	

**Table 3**

**Solution 2**

Retail shops					
Factories	1	2	3	4	Supply
1	3 <sup>20</sup>	5 <sup>20</sup>	7 <sup>10</sup>	6	50
2	2	5	8 <sup>40</sup>	2 <sup>35</sup>	75
3	3	6	9	2 <sup>25</sup>	25
Demand	20	20	50	60	

**Table 4**

Starting from the North west corner, we allocate  $x_{11} = 20$ . Now demand for the first column is satisfied, therefore, eliminate that column.

Proceeding in this way, we observe that  $x_{12} = 20$ ,  $x_{13} = 10$ ,  $x_{23} = 40$ ,  $x_{24} = 35$ ,  $x_{34} = 25$ .

**Delete the row if supply is exhausted.**  
**Delete the column if demand is satisfied.**

Here, number of retail shops( $n$ ) = 4, and  
 Number of factories ( $m$ ) = 3

Number of basic variables =  $m + n - 1 = 3 + 4 - 1 = 6$ .

**Initial basic feasible solution:**

$$20 * 3 + 20 * 5 + 10 * 7 + 40 * 8 + 35 * 2 + 25 * 2 = 670$$

**2. Minimum Matrix Method (MMM)**

Matrix minimum method is a method for computing a basic feasible solution of a transportation problem where the basic variables are chosen according to the unit cost of transportation.

**Steps**

1. Identify the box having minimum unit transportation cost ( $c_{ij}$ ).
2. If there are two or more minimum costs, select the row and the column corresponding to the lower numbered row.
3. If they appear in the same row, select the lower numbered column.
4. Choose the value of the corresponding  $x_{ij}$  as much as possible subject to the capacity and requirement constraints.
5. If demand is satisfied, delete the column .
6. If supply is exhausted, delete the row.
7. Repeat steps 1-6 until all restrictions are satisfied.

**Example 3**

Retail shops					
Factories	1	2	3	4	Supply
1	3	5	7	6	50
2	2	5	8	2	75
3	3	6	9	2	25
Demand	20	20	50	60	

**Table 5**

**Solution 3**

Retail shops					
Factories	1	2	3	4	Supply
1	3	5 <sup>20</sup>	7 <sup>30</sup>	6	50
2	2 <sup>20</sup>	5	8	2 <sup>55</sup>	75
3	3	6	9 <sup>20</sup>	2 <sup>5</sup>	25
Demand	20	20	50	60	

**Table 6**

We observe that  $c_{21} = 2$ , which is the minimum transportation cost. So,  $x_{21} = 20$ .

Proceeding in this way, we observe that  $x_{24} = 55$ ,  $x_{34} = 5$ ,  $x_{12} = 20$ ,  $x_{13} = 30$ ,  $x_{33} = 20$ .

Number of basic variables =  $m + n - 1 = 3 + 4 - 1 = 6$ .

**The initial basic feasible solution:**

$$20 * 2 + 55 * 2 + 5 * 2 + 20 * 5 + 30 * 7 + 20 * 9 = 650.$$

**3. Vogel's Approximation Method (VAM)**

The Vogel approximation method is an iterative procedure for computing a basic feasible solution of the transportation problem.

**Steps**

1. Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.

2. Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column
3. Identify the maximum penalty. If it is along the side of the table, make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.
4. If the penalties corresponding to two or more rows or columns are equal, select the top most row and the extreme left column.

**Example 4**

Consider the transportation problem presented in table7 :

<b>Destination</b>					
<b>Origin</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>Supply</b>
<b>1</b>	20	22	17	4	120
<b>2</b>	24	37	9	7	70
<b>3</b>	32	37	20	15	50
<b>Demand</b>	60	40	30	110	240

**Table 7**

### Solution 4

Destination						
Origin	1	2	3	4	Supply	Penalty
1	20	22 <sup>40</sup>	17	4	<del>120</del> 80	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
<b>Demand</b>	60	40	30	110	240	
<b>Penalty</b>	4	15	8	3		

**Table 8**

The highest penalty occurs in the second column. The minimum  $c_{ij}$  in this column is  $c_{12}$  (i.e., 22). Hence,  $x_{12} = 40$  and the second column is eliminated.

Now again calculate the penalty

Destination						
Origin	1	2	3	4	Supply	Penalty
1	20	22 <sup>40</sup>	17	4 <sup>80</sup>	<del>120</del>	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
<b>Demand</b>	60	40	30	110	240	
<b>Penalty</b>	4	-	8	3		

**Table 9**

The highest penalty occurs in the first row. The minimum  $c_{ij}$  in this row is  $c_{14}$  (i.e., 4). So  $x_{14} = 80$  and the first row is eliminated.

**Final table**

Now we are assuming that you can calculate the values yourself.

Destination											
Origin	1	2	3	4	Supply	Penalty					
1	20	22 <sup>40</sup>	17	4 <sup>80</sup>	120	3	13	-	-	-	-
2	24 <sup>10</sup>	37	9 <sup>30</sup>	7 <sup>30</sup>	70	2	2	2	17	24	24
3	32 <sup>50</sup>	37	20	15	50	5	5	5	17	32	-
<b>Demand</b>	60	40	30	110	240						
<b>Penalty</b>	4	15	8	3							
	4	-	8	3							
	8	-	11	8							
	8	-	-	8							
	8	-	-	-							
	24	-	-	-							

**Table 10**

**The initial basic feasible solution:**

$$22 * 40 + 4 * 80 + 24 * 10 + 9 * 30 + 7 * 30 + 32 * 50 = 3520$$