Unit 1

Lesson 7: Simplex Method Ctnd.

Learning outcomes:

- **Set up and solve LP problems with simplex tableau.**
- **Interpret the meaning of every number in a simplex tableau.**

Dear students, today we discuss small cases or case-lets on the above topic. The situations presented below are real business problems, modified slightly to suit our purpose.

We start now.

Case-let-1

A firm makes air coolers of three types and markets these under the brand name "Symphony".
The relevant details are as follows:-

Profit / unit (Rs.)	300 Product A (hrs. / unit)	700 Product B (hrs. / unit)	900 Product C (hrs,/ unit)	Totoal hrs. available
Designing	0	10	20	320
Manufacturing	60	90	120	1600
Painting	30	40	60	1120

What Qty. of each product must be made to maximize the total profit of the firm?

Solution

Let X_1, X_2, X_3 denote the Qty. of each product made

Then: Maximize: $300 X_1 + 700 X_2 + 900 X_3$

Subject to: $0 X_1 + 10 X_2 + 20 X_3 \leq 320$

$60 X_1 + 90 X_2 + 120 X_3 \leq 1600$

$30X_1 + 40X_2 + 60X_3 \leq 1120$

Introducing slacks:

Maximize: $300X_1 + 700X_2 + 900X_3 + 0S_1 + 0S_2 + 0S_3$

Subject to: $0X_1 + 10X_2 + 20X_3 + S_1 + 0S_2 + 0S_3 = 320$

$60X_1 + 90X_2 + 120X_3 + 0S_1 + S_2 + 0S_3 = 1600$

$30X_1 + 40X_2 + 60X_3 + 0S_1 + 0S_2 + S_3 = 1120$

1st Tableau

C_J(Rs.) (Rs.)	Basic Act.	Qty.	300 X_1	700 X_2	900 X_3	0 S_1	0 S_2	0 S_3
0	S_1	320	0	10	20	1	0	0
0	S_2	1600	60	90	120	0	1	0
0	S_3	1120	30	40	60	0	0	1
	Z_J	0	0	0	0	0	0	0
	$C_J - Z_J$		300	700	900	0	0	0

Pivot column = X_3

Pivot –row = $S_2 \left[S_1 : \frac{320}{20} = 16, S_2 : \frac{1600}{120} = \frac{40}{3}, S_3 : \frac{1120}{60} = \frac{56}{3} \right]$

Pivot element = 120

Pivot row updating : $\frac{1600}{120} = \frac{40}{3}, \frac{60}{120} = \frac{1}{2}, \frac{90}{120} = \frac{3}{4}, 1, 0, \frac{1}{120}, 0$

Up-dating S_1 row:

$320 - \left(20x\frac{40}{3}\right) = \frac{160}{3}$

$0 - \left(120X\frac{1}{2}\right) = -10$

$20 - \left(20x\frac{3}{4}\right) = -5$

$1 - (20x0) = 5$

$0 - \left(20x\frac{1}{120}\right) = \frac{-1}{6}$

$1 - (20x0) = 0$

Updating S_3 row:

$\frac{1}{20} - \left(60x\frac{40}{3}\right) = 320$

$30 - \left(60X\frac{1}{2}\right) = -0$

$40 - \left(60X\frac{3}{4}\right) = -5$

$0 - (60x0) = 0$

$0 - \left(60x\frac{1}{120}\right) = \frac{-1}{2}$

$1 - (60x0) = 1$

IInd Tableau

Cj (Rs.) (Rs.)	Basic Act.	Qty.	300 x_1	700 x_2	900 X_3	0 S_1	0 S_2	0 S_3
0	S_1	160/3	-10	-5	0	1	-1/6	0
900	X_3	40/3	½	¾	1	0	1/120	0
0	S_3	320	0	-5	0	0	-1/2	1
	Zj (Rs.)	2000	450	675	900	0	15/2	0
	Cj-Zj (Rs.)		-150	25	0	0	-15/2	0

Since, There is still one + ve term in row, pivoting is requd.
Pivot – Column = x_2
Pivot – row =

$$x_3\left[S_1 : \frac{160}{3} = -5\,(Not\ considere3d), x_3 : \frac{40}{3} \div \frac{3}{4} = \frac{160}{9}, S_3 : \frac{320}{5}(Notconsi.)\right]$$

Pivot – element =3/4
Pivot-row updating:

$$\frac{40}{3} \div \frac{3}{4} = \frac{160}{9}, \frac{1}{2} \div \frac{3}{4} = \frac{2}{3}, 1, 1 \div \frac{3}{4} = \frac{4}{3}, 0, \frac{1}{120} \div \frac{3}{4} = \frac{1}{90}, 0$$

Updating S_1 row:

$$\frac{160}{3} - \left(-5x\frac{160}{9}\right) = \frac{1280}{9}$$
$$-10 - \left(-5x\frac{2}{3}\right) = \frac{20}{3}$$
$$-5 - (5\,x\,1) = 0$$
$$0 - \left(-5\,x\frac{4}{3}\right) = \frac{20}{3}$$
$$1 - (-5\,x0) = 1$$
$$-\frac{1}{6} - \left(-5\ x\frac{1}{90}\right) = \frac{-1}{9}$$
$$0 - (-5\,x0) = 0$$

Updating S_3 row:

$$320 - \left(-5x\frac{160}{9}\right) = \frac{3680}{9}$$
$$0 - \left(-5x\frac{2}{3}\right) = \frac{10}{3}$$
$$-5 - (-5\,x\,1) = 0$$
$$0 - \left(-5\,x\frac{4}{3}\right) = \frac{20}{3}$$
$$0 - (-5\,x0) = 0$$
$$-\frac{1}{6} - \left(-5\ x\frac{1}{90}\right) = \frac{-4}{9}$$
$$1 - (-5\,x0) = 1$$

IIIrd Tableau

Cj (Rs.) (Rs.)	Basic Act.	Qty.	300 x_1	700 x_2	900 x_3	0 S_1	0 S_2	0 S_3
0	x_1	1280/9	-20/30	0	20/3	1-	1/9	0
700	S_2	160/9	2/3	1	4/3	0	1-90	0
0	S_3	3680/9	10/3	0	20/3	0	-4/9	1
	Zj (Rs.)	112000	1400/3	700	2800/3	0	70/9	0
	Cj-Zj (Rs.)		-500/3	0	-100/3	0	-70/9	0

Since Cj – Zj row has no. + ve value left.
∴ It is the optional soln.

Case-let-II

Two materials A & B are required to construct table & book cases. For one table, 12 units of A & 16 units of B are required while for a book case 16 units of A and 8 units of B are required. The profit on book case is Rs 25 and Rs 20 on a table. 100 units of A & 80 units of B are available. Formulate as a Linear programming problem & determine the optimal number of book cases & tables to be produced so as to maximise the profits.

Solution

Let x_1 = no. of unit of tables to be produced, and
X_2 = no. of unit of book cases to be produced.

Formulating as a LP problem, we have:-

Maximise $Z = 20x_1 + 25x_2$ (objective function)

Subject to the constraints: $12x_1 + 16x_2 \leq 100$

$16x_1 + 8x_2 \leq 80$

$x_1, x_2 \geq 0$

Introducing the slack variable, we have:

$12x_1 + 16x_2 + S_1 \leq 100$

$16x_1 + 8x_2 + S_2 \leq 80$

Re-writing as:

$$\begin{bmatrix}12\\16\end{bmatrix}x_1+\begin{bmatrix}16\\8\end{bmatrix}x_2+\begin{bmatrix}1\\0\end{bmatrix}S_1+\begin{bmatrix}1\\100\end{bmatrix}S_2=\begin{bmatrix}100\\80\end{bmatrix}$$

or $\quad P_1x_1+P_2x_2+P_3S_1+P_4S_2=Po$

$\therefore$ our problem becomes Maximise

Subject to $F=20x_1+25x_2+0\times S_1+0\times S_2$ …(1)

Where $P_1x_1+P_2x_2+P_3S_1+P_4S_2=Po$ …(ii)

$$P_0\begin{bmatrix}100\\80\end{bmatrix}P_2+\begin{bmatrix}1\\0\end{bmatrix}P_4+\begin{bmatrix}1\\0\end{bmatrix}P_2=\begin{bmatrix}13\\16\end{bmatrix}P_2=\begin{bmatrix}16\\8\end{bmatrix}$$

element of Cj row are values of P_o, P_2,P_4,P_1, in (i) by comparing with (2) i.e. elements of Cj are 0, 0, 20, 25.

Simplex Method

	Cj		*0*	*0*	*0*	*20*	*25*	*Ratio*
		Vectors	P_o	P_2	P_4	P_1	P_3	
R_1	0	P_3	*100*	*1*	*0*	*12*	*16*	*100/16=6.25*
R_2	0	P_4	*80*	*0*	*1*	*16*	*0*	*80/8=10*
Stage I	Zj		*0*	*0*	*0*	*0*	*0*	*6.25 is least ratio*
	Zj – Cj		*0*	*0*	*0*	*20*	*25*	$\therefore$ *replaced vector is* P_2
		R_1 *is*	$\frac{R_1}{16}$ $R_2 = R_2 - R_1 \times 8/16$					Since 25 is the most postive No. in row $C_4 - Z_4$ $\therefore$ Replacing vector is P_2
R_1	25	P_2	$\frac{25}{4}$	*1/16*	*0*	$\frac{3}{4}$	1	$\frac{25}{4}/\frac{5}{4}=\frac{25}{3}$ =8.33
R_2	0	P_4	*30*	$-\frac{1}{3}$	*1*	*10*	0	*30/10=3*
Stage I	Zj		$\frac{625}{4}$	$\frac{25}{16}$	*0*	$\frac{75}{4}$	25	Θ 3 is least ratio $\therefore$ replaced vector is P_4
	Zj – Cj		$\frac{625}{4}$	$\frac{25}{16}$	*0*	$-\frac{5}{4}$	0	Θ $-\frac{5}{4}$ is least – ve no. $\therefore$ replaced vector is P_1

$$R_2\ is\ \frac{R_1}{10}$$

$$R_1 = R - R_2\ \frac{3/4}{10} = R_1 - R_2\frac{3}{40}$$

The elements of column Cj are values of P_3 and P_4 is (I) as compared with (2). The column P_0,P_2,P_4,P_1,P_3 are values of P_0,P_2,P_1 etc. in (3)

R_1	25	P_2	4	1/10	/3/40	0	1
R_2	20	P_4	3	-1/20	1	1	0
Stage I	Zj		160	1.5	1/8	20	25
	Zj – Cj	160	1.5	1/8	0	0	

Since in stage III all elements of row Zj – Cj are +ve or zero, hence an optimal solution has been achieved.

The solution is given by column is Stage III.

$\therefore P_0 = 3P_1 + 4P_2$

compare it with $P_1x_1 + P_2x_2 + P_3S_1 + P_4S_2 = P_0$

$$x_1 = 3, x_2 = 4, S_1 = 0, S_2 = 0$$

Note I. the elements of row Zj is the sum of product of corresponding elements of column Cj with P_o, P_3, P_4, P_1, P_2 respectively.

For example in stage II, the elements of row Zj are

$$25x\frac{25}{4} + 0x30 = \frac{625}{4}$$

$$25x\frac{1}{16} + 0x = \left(\frac{-1}{2}\right)\frac{25}{16}$$

25x 0+0 x 1=0

$$25x\frac{3}{4} + 0x10 = \frac{75}{4}$$

25 x 1 + 0x 0 =25.

Note 2. When we replace vector P_2 in place of P_3 in stage II, the elements to left of P_3 under column Cj is also changed.

Note. To check the results.

At x=3, $x_2 = 4$

Constraints are $12x_1 + 16x_2 = 36 + 64 = 100 \le 100$

$16x_1 + 8x_2 = 48 + 32 = 80 \le 80$

Case-let-III

A manufacturer produces children's bicycles & scooters both of which one processed through two machines. Machine 1 has a maximum of 120 hours available & machine 2 has a maximum of 180 hours. Manufacturing a bicycle requires 6 hours on machine 1 x 4 hours on machine 2. A scooter requires 3 hours on machine 1 & ten hours on machine 2. If the profit is Rs 45 on a bicycle & Rs. 55 on a scooter determine the number of bicycles & scooters that should be produced in order to maximise profit.

Solution

Let no. of bicycles and scooters produced by x and y units respectively.
$\therefore$ L.P. problem is Maximize P = 45x + 55y
Subject to $6x + 3y \le 120$
$4x + 10y \le 180,\ x \ge 0, y \ge 0$

Introducing the slack variables, we get
$6x + 3y_2 + S_1 + 120$
$4x + 10y_2 + S_2 + 180$
writing it in vector form

$$\begin{bmatrix} 6 \\ 4 \end{bmatrix} x + \begin{bmatrix} 3 \\ 4 \end{bmatrix} y + \begin{bmatrix} 1 \\ 0 \end{bmatrix} S_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} S_2 = \begin{bmatrix} 120 \\ 180 \end{bmatrix} \qquad \ldots(1)$$

or $P_1x+P_2y+p_2S_1+P_4S_2=P_0$
Θ Our L.P.P is Max, $P=45x+55y+OS_1+OS_2$
Θ Subject to $P_1x + P_2y+P_3S_1 + P_4S_2=P_o$ …(2)
Comparing (1) and (3), we have

$$P_1 = \begin{bmatrix} 6 \\ 4 \end{bmatrix}' P_2 = \begin{bmatrix} 3 \\ 10 \end{bmatrix}, P_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, P_4 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$P_o = \begin{bmatrix} 120 \\ 180 \end{bmatrix}'$$

The elements of row are value of
Po, P2,P4,P1,P3, in (1) by comparing it with (2)
i.e., element of row are o, o, 45,45.,55.
Note. The element of row are sum of product of corresponding element of columns and column vectors

Simplex Method

	Cj		*0*	*0*	*0*	*45*	*55*	*Ratio*
		Vectors	*P_o*	*P_3*	*P_4*	*P_1*	*P_2*	
R_1	0	*P_3*	*120*	*1*	*0*	*6*	*3*	
R_2	0	*P_4*	*180*	*0*	*1*	*4*	*10*	$\alpha 40/\alpha 41 = 180/4 = 45$
Stage I	Zj		*0*	*0*	*0*	*0*	*0*	$\therefore$ *20 is least ratio, so*
	Z_j-C_j		*0*	*0*	*0*	*-45*	*-55*	*replace vector is P_3* Θ - 55 is least no. in row Z_j-C_j, $\therefore$ *replacing vector is P_1*

Row $R_1 \dfrac{R_1}{a_{31}} = \dfrac{R_1}{6}$

Row $R_2 = R_2 - R_1 \times \frac{a_{41}}{a_{31}} R_2 - R_1 \times \frac{4}{6}$.

Further calculations are left to the students as an exercise.

Case-let-IV

.A firm has the following availabilities :

Type-available	Amount-available (kg)
Wood	240
Plastic	370
Steel	180

The firm produces two products A & B having a selling price of Rs. 4 per unit & Rs. 6 per unit respectively. The requirements for the manufacture of A & B are as follows:

Product	Requirements of (kg)		
	Wood	Plastic	Steel
A	1	3	2
B	3	4	1

Formulate as a LP problem & solve by using the simplex method to maximise the gross income of the firm.

Case-let-V

Ace- advantage Ltd. faces the following situation:
Media available - electronic (A) & print (B)
Cost of available in - media A: Rs.1000
Media B: Rs.1500
Annual advertising budget - Rs. 20000

The following constraints are applicable:
Electronic Media (A) can not have more than 12 advertisements in a year and not less than 5 advertisement must be placed in the print media (B).

The estimated audience are as follows:
Electronic media (A) - 40000
Print media (B) - 55000
You are required to develop a mathematical model & solve it for maximizing the total effective audience.

Case-let-VI

Khalifa & sons sells two different books B1 & B2 at a profit margin of Rs. 7 and Rs. 5 per book respectively. B1 requires 5 units of raw material & B2 requires 1 units of raw material. The maximum availability of raw materials is limited to 15units. To maintain the high quality of books, it is desired to follow the given quality constraint: $3x_1 + 7x_2 \geq 21$. Formulating

As a LP model determine the optimal solution.

Dear students, we have now reached the end of our discussion scheduled for today.
See you all in the next lecture.
Bye.