

# Unit 1

## Lesson 6: Simplex Method

- **Set up and solve LP problems with simplex tableau.**
- **Interpret the meaning of every number in a simplex tableau.**

**Dear Students, all of us have by now mastered the graphical method of SOLVING A LINEAR PROGRAMMING MODEL**

**Well friends, let us now focus on the LIMITATIONS OF THE GRAPHICAL METHOD OF SOLVING A LINEAR PROGRAMMING MODEL.**

Let us see what the limitations are, and how can these be tackled?  
Here we go.

### **LIMITATIONS OF THE GRAPHICAL METHOD**

**Well friends, once a Linear programming model has been constructed on the basis of the given constraints & the objective function, it can easily be solved by using the graphical method (as discussed in earlier lectures) & the optimal solution can be generated.**

However, the applicability of the graphical method is very limited in scope. This is due to the fact that it is quite simple to identify all the corner points & then test them for optimality-in the case of a two-variable problem. As a result, the graphical method can not be always employed to solve the real-life practical Linear programming models which involve more than two decision-variables.

The above limitation of the graphical method is tackled by what is known as the simplex method. Developed in 1947 by George B-Dantizg, it remains a widely applicable method for solving complex LP problems. It can be applied to any LP problem which can be expressed in terms of a Linear Objective function subject to a set of Linear Constraints. As such, no theoretical restrictions are placed on the number of decision variables or constraints contained in a linear programming problem.

The simplex method employed in solving LP problem is discussed as under:

### **The simplex method**

The graphical method, as discussed in the previous lectures, is capable of solving problems having a maximum of two variables. Hence, this method is used which can solve LP problems with any no. of variable or constraints it is geared towards

solving optimization problems which have constraints of less than or equal to type.

- (i) This method utilizes the property of a LP problem of having optimal solution only' at the corner point of the feasible solution space. It systematically generates corner point solutions & evaluates them for optimality. The method stops when an optimal solution is found. Hence, it is an iterative (repetitive) technique.

If we get more variables & less equations, we can set extra variables equal to zero, to obtain a system of equal variables & equal equations. Such solution is called basic solution.

- (ii) The variables having positive values in a basic feasible solution are called basic variable while the variables which are set equal to zero, so as to define a corner point are called non-basic variables.
- (iii) Slack variables are the fictitious variables which indicate how much of a particular resource remains unused in any solution. These variables can not be assigned negative values. A zero value indicates that all the resources are fully used up in the production process.
- (iv)  $C_j$  column denotes the unit contribution margin.
- (v)  $C_j$  row is simply a statement of the projective function.
- (vi)  $Z_j$  row denotes the contribution margin lost if one unit is brought into the solution. Hence, it represents the opportunity cost. (Opportunity cost is the cost of sacrifice i.e., the opportunity foregone by selecting a particular course of action out of a number of different available alternatives).
- (ix)  $C_j - Z_j$  row denotes the Net Potential contribution or the Net unit Margin potential, per unit.
- (vii) The rules used under simplex method, for solving a linear programming problem are as follows:-

### 1. Convert the LP to the following form:

Convert the given problem into Standard maximization Problem i.e. minimization problem into a maximization one (by multiplying the objective function by -1).  
All variables must be non-negative.

All RHS values must be non-negative (multiply both sides by -1, if needed).

All constraints must be in  $\leq$  form (except the non-negativity conditions). No strictly equality or  $\geq$  constraints are allowed.

2. **Convert all  $\leq$  constraints to equalities by adding a different slack variable for each one of them.**
  
3. **Construct the initial simplex tableau with all slack variables in the BVS.** The last row in the table contains the coefficient of the objective function (row  $C_j$ ).
  
4. **Determine whether the current tableau is optimal.** That is:  
If all RHS values are non-negative (called, the feasibility condition)  
If all elements of the last row, that is  $C_j$  row, are non-positive (called, the optimality condition).  
  
If the answers to both of these two questions are yes, then stop. The current tableau contains an optimal solution.  
Otherwise, go to the next step.
  
5. **If the current BVS is not optimal, determine, which non basic variable should become a basic variable and, which basic variable should become a non basic variable.** To find the new BVS with the better objective function value, perform the following tasks:
  - **Identify the entering variable:** The entering variable is the one with the largest positive  $C_j$  value (In case of a tie, select the variable that corresponds to the leftmost of the columns).
  - **Identify the outgoing variable:** The outgoing variable is the one with smallest non-negative column ratio (to find the column ratios, divide the RHS column by the entering variable column, wherever possible). In case of a tie select the variable that corresponds to the up most of the tied rows.

## Generate the new tableau

(a) Select the largest value of  $C_j - Z_j$  row. The column, under which this value falls is the pivot-column.

(b) Pivot-row selection rule. Find the ratio of quantity to the corresponding pivot-column co-efficient. The pivot-row selected "is the variable having the least ratio.

**Remarks. Rows having negative or zero co-efficient in the pivot-column are to be neglected.**

(c) The coefficient, which is in both, the pivot-row & the pivot-column is called the pivot-element or pivot-no.

(d) Up-dating Pivot-row. Pivot-row, also called replaced rows, are updated as under.

All elements of old-row divided by Pivot-element

Now, in the basic activities column, write the pivot-column variable in place of the pivot-row variable. i.e.; the pivot-row variable is to be replaced by the pivot-column variable.

(e) Up-Dating all other rows. Up date all other rows by updating the formulae.

**(Old-row element) - (Corresponding pivot column element \* updated corresponding pivot row element) = (New element)**

(f) Up-dating  $Z_j$  &  $C_j - Z_j$  rows. Each  $Z_j$ , is obtained as the sum of the products of the  $C_j$  column coefficients multiplied by the corresponding coefficient in the  $J$ th column. (i.e.) the Quantity column).

It is then subtracted from  $C_j - Z_j$  row values to get  $C_j - Z_j$  values.

**This pivoting is to be repeated till no positive coefficients exist in the  $C_j - Z_j$  row, the optimal solution is known.**

## What is a Standard Maximization Problem?

A Standard Maximization Problem is the one that satisfies the following 4 conditions

1. The Objective function is to be maximized.
2. All the inequalities are of  $\leq$  type.
3. All right hand constants are non-negative.
4. All variables are non-negative.

Friends, let us consider some examples to test our understanding of the solution algorithm that has been discussed so far.

**Example 1.**

**Smart Limited manufactures two types of adhesives which are sold under the brand name quick and Tuff. Each product consumes the same raw materials but in varying proportions. The following table depicts the amount of raw materials along with their respective cost.**

Raw material type	Price (Rs/ltan)	Quick	Tuff
N	600	350	200
A	400	50	100
P	400	50	100
I	200	550	600
		1000 kg	1000 kg

Quick can be blended @ 1000 kg /hour whereas the blending rate for Tuff is 1250 kg /hour. Their respective selling prices are Rs. 1010 & Rs. 845. You may assume the variable costs to be Rs. 500 per hour of plant production time. The maximum availability of raw materials is:

Raw Material Types	Max available (kg)
N	1000
A	300
P	250
I	1800

Formulate as a linear programming model and find out the optimal units of quick & tuff to be produced so as to maximise the profits.

**Solution**

**Step I**

List the objective & constraint equations.

**Step II**

Introduce the slack variables

**Step III**

Arrange in the form of 1<sup>st</sup> tableau (table)

**Step IV**

Find out the profit-margins from given sales price.

**Step V**

Generate solutions.

The detailed solutions are as under:

**Simplex Method**

(I) Converting in equations into equations by using slack-variable

Maximise Cont-margin =  $150x_1 + 125x_2$

Subject to  $350x_1 + 200x_2 + S_1 = 1000$

$50x_1 + 100x_2 + S_2 = 250$

$550x_1 + 600x_2 + S_3 = 1800$

$S_i, x_j \geq 0$  for all  $i$  &  $j$ .

Note. We have dropped :  $50x_1 + 100x_2 \leq 300$  as it is already contained in

$50x_1 + 100x_2 \leq 250$

Also, slack can't be larger than the const. On RHS

(  $S_1 = 1000, S_2 = 250$  etc) It is were to be so, then some other variable must be negative for equality to exist, which is not possible.

(2) Rewriting as:

Maximising cost. Margin -  $150x_1 + 125x_2 + OS_1 + OS_2 + OS_3$

Subject to  $350x_1 + 200x_2 + S_1 + OS_2 + OS_3 = 1000$

$50x_1 + 100x_2 + OS_1 + OS_2 + OS_3 = 250$

$550x_1 + 600x_2 + OS_1 + OS_2 + OS_3 = 1800$

$S_i, x_j \geq 0$  & (  $i, j$  )

(3) Arranging in tableau-form:

Cj (Rs.)	Basic Activities	(Rs.) Qty.	150 $x_1$	125 $x_2$	0 $S_1$	0 $S_2$	0 $S_3$
0	$S_1$	1000	350	200	1	0	0
0	$S_2$	250	50	100	0	1	0
0	$S_3$	1800	50	600	0	0	1
	ZJ(Rs.)	0	0	0	0	0	0
	Cj-Zj (Rs.)		150	125	0	0	0

(4) Pivot Column =  $x_1$  ( Largest value of  $C_j - Z_j$  )

(5) Pivot row

$$S_1, \frac{1000}{350} = 2.857 \text{ (smallest value)}; S_2, \frac{350}{50} = 5; S_3, \frac{1800}{550} = 3.273$$

$\therefore S_1$  is the pivot- row

Pivot element = 350

### How to calculate the Profit Margin?

Let us first find the time taken to manufacture 1000 kg of both types. It is required for allocating variable production costs to finished product.

$\therefore$  1250 kg Tuff is made in one hour.

$\therefore$  100 kg = 0.8 hr.

$\therefore$  Variable production cost for Tuff  $0.8 \times 500 = \text{Rs. } 400$

$\therefore$  Variable production cost for Quick Rs. 500 ( $\therefore$  In 1 hr. 1000 kg is made)

<i>Particulars Revenue/sales (given)</i> <i>(-) var costs: Direct Mat</i>	<i>Quick (Rs.)</i> <i>1,0,10</i>	<i>Tuff (Rs.)</i> <i>845</i>
N –	310	120
A –	20	40
P –	110	120
I –	20	40
Sub-total	360	320
Direct paid cost	500	400
Total var. cost	860	720
Contribution margin	150	125

### Remarks

- Given cost of 1000 kg of No. Rs. 600  
Given cost of 350 kg of No. = Rs. 210 ( for Quick )  
Given cost of 200 kg of No. Rs. 120 ( for Tuff) etc.
- Contribution margin represents the profit which remains if after deducting var costs from sales i.e.,; It covers fixed costs & net profit i.e. it fixed cost = Rs. 200 X five – 100 kg of quick are sold then the net profit =  $5 \times 150 - 200 = \text{Rs. } 550$   
( F. cost donot vary with the output )
- All calculation has been done to obtain the cont. margin ( i.e., profit )  
let  $x_1$  = thousands of kg of quick to be produced.  
 $x_2$  = thousands of kg of tuff to be produced.

Now, we have to find those values of  $x_1$  &  $x_2$  for which the contribution is max.

Here, the constraint is the availability of raw mats. Hence, the problem is formulate as;

Maxmise  $150x_1 + 125x_2$   
 Subject to:

$$\left. \begin{aligned} 350x_1 + 200x_2 &\leq 1000 \\ 50x_1 + 100x_2 &\leq 300 \\ x_1, x_2 &\geq 0 \end{aligned} \right\} \text{Hence, the problem has been}$$

formulated

$$550x_1 + 600x_2 \leq 1800$$

(6) up-dating Pivot-row

$$\frac{1000}{350} = 2.8571, \frac{350}{350} = 1, \frac{200}{350} = 0.5714, \frac{1}{350} = 0.0029, 0, 0$$

Make the pivot-row of 2<sup>nd</sup> tableau:

$C_j$			150	125	0	0	0
	Basic Act.	Qty.	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$
150	$x_1$	2.8571	1	0.5714	0.0029	0	0
0	$S_2$						
0	$S_3$						

(Pivot column variable in place of Pivot row variable)

(7)  $S_2$  row is up-dated as follows:

Old Row Element -	(Corresp. Pivot column element $x$ )	Updated corresp. Pivot row element	= New $S_2$ raw element
250 -	( 50 x	2.8571)	= 107.145
50 -	( 50 x	1)	= 0
100 -	( 50 x	0.5714	= 71.43
0 -	( 50 x	0.0029)	= -0.145
1 -	( 50 x	0)	= 1
0 -	( 50 x	0)	= 0
	( 50 x		

(8)  $S_3$  row is similarly updated:

Old Row Element -	(Corresp. Pivot column element $x$ )	Updated corresp. Pivot row element	= New $S_2$ raw element
1800 -	( 50 x	2.8571)	= 228.595
550 -	( 550 x	1)	= 0
600 -	( 550 x	0.5714)	= 285.73
0 -	( 550 x	0.0029)	= -1.595
0 -	( 550 x	0)	= 0
1 -	( 550 x	0)	= 1
	( 550 x		



(9) Complete IIInd tableau is:

Cj (Rs)	Basic Activity	(Rs.) Qty.	1.50 x <sub>1</sub>	125 x <sub>2</sub>	0 S <sub>1</sub>	0 S <sub>2</sub>	0 S <sub>3</sub>
150	X <sub>1</sub>	2.857	1	0.5714	0.0029	0	0
0	S <sub>2</sub>	107.145	0	71.43	-0.145	1	0
0	S <sub>3</sub>	228.595	0	285.73	1.595	0	1
-	Zj (Rs.)	428.55	150	85.71	0.435	0	0
-	Cj-Zj (Rs.)	--	0	39.29	-0.435	0	0

Zj is calculated as under:

Zj value for QTY Column =  $150 \times 2.857 + 0 \times 107.145 + 0 \times 228.595 = 428.55$

For  $x_1 = 150 \times 1 + 0 \times 0 + 0 \times 0 = 150$

$x_2 = 150 \times 0.5714 + 0 + 0 = 85.71$

$S_1 = 150 \times 0.0029 + 0 + 0 = 0.435$

$S_2 = 0$

$S_3 = 0$

Now perform Cj – Zj to get Cj – Zj row values.

(10) The + ve value of 39.29 in Cj – Zj -> It is not the optimal soln Hence, once again pivoting is required.

Developing IIIrd tableau.

Now, pivot column – x<sub>2</sub> ( Largest value of Cj – Zj ), Pivot row = S<sub>3</sub>

Pivot element = 285.73

Updating Pivot row ( S<sub>3</sub>; Old row element ÷ Pivot no.

$$\frac{228.595}{285.73} = 0.8, \quad 0, \quad \frac{285.73}{283.73} = 1, \quad \frac{-1.595}{285.73} = -0.0056, \quad 0,$$

$$\frac{1}{285.73} = 0.0035$$

**Completed IIIrd Tableau is.**

Cj (Rs)	Basic Activity	(Rs.) Qty.	150 x <sub>1</sub>	125 x <sub>2</sub>	0 S <sub>1</sub>	0 S <sub>2</sub>	0 S <sub>3</sub>
150	x <sub>1</sub>	2.4	1	0	0.006	0	-0.002
0	S <sub>2</sub>	50	0	0	0.225	1	-0.25
125	x <sub>2</sub>	0.8	0	1	-0.0056	0	0.0035
	Zj (Rs.)	460	150	125	0.02	0	0.1375
	Cj-Zj (Rs.)		0	0	-0.02	0	-0.1375

**Note**

- (1) x<sub>1</sub> row is up-dated as:  
 $2.857 - (0.5714 \times 0.8) = 2.4$   
 $1 - (0.5714 \times 0) = 1$

$$\begin{aligned}
& 0.5714 - (0.5714 \times 1) = 0 \\
& 0.0029 - (0.5714 \times -0.0056) = 0.006 \\
& 0 - (0.5714 \times 0) = 0 \\
& 0 - (0.5714 \times 0.0035) = -0.002 \\
(2) \quad & S_2 \text{ row is up dated as:} \\
& 107.145 - (71.43 \times 0.8) = 50 \\
& 0 - (71.43 \times 0) = 0 \\
& 71.43 - (71.43 \times 1) = 0 \\
& -0.145 - (71.43 \times -0.0056) = 0.255 \\
& 1 - (71.43 \times 0) = 1 \\
& 0 - (71.43 \times 0.0035) = -0.25
\end{aligned}$$

4	$X_2$	2100	3/10	1	0	0	1/10
	$Z_j$ (Rs.)	12300	+1/10	4	0	0	3/10
	$C_j - Z_j$ (Rs.)	-1/10		0	0	0	-3/10

<p><b>Up-dating <math>S_1</math> row</b></p> $600 - \left(\frac{4}{13} \times 1300\right) = 200$ $\frac{4}{13} - \left(\frac{4}{13} \times 1\right) = 0$ $0 - \left(\frac{4}{13} \times 0\right) = \frac{-4}{13}$ $0 - \left(\frac{4}{13} \times \frac{-13}{30}\right) = \frac{2}{15}$ $-1 - \left(\frac{4}{13} \times 0\right) = -1$ $-\frac{1}{13} - \left(\frac{4}{13} \times \frac{-1}{30}\right) = \frac{-1}{15}$	<p><b>Updating <math>x_2</math> row</b></p> $3000 - \left(\frac{9}{13} \times 1300\right) = 2100$ $\frac{9}{13} - \left(\frac{9}{13} \times 1\right) = 0$ $1 - \left(\frac{9}{13} \times 0\right) = 1$ $0 - \left(\frac{9}{13} \times \frac{-13}{30}\right) = \frac{3}{10}$ $0 - \left(\frac{9}{13} \times 0\right) = 0$ $\frac{1}{13} - \left(\frac{9}{13} \times \frac{-1}{13}\right) = \frac{1}{10}$
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**Up dating  $z_j$  row:  $z_j$  values for:**

$$Qty = 3 \times 1300 + 4 \times 2100 = 12300$$

$$X_1 = 3$$

$$X_2 = 4$$

$$S_2 = \frac{-39}{30} + \frac{12}{10} = \frac{-1}{10}$$

$$S_2 = 0$$

$$S_1 = +\frac{4}{10} = \frac{3}{10}$$

Since no positive co-efficient exists in the  $C_1 - z_1$  row, this is the optimal solution.

$X_1 = 1300 =$  maximum pens to be manufactured

$X_2 = 2100 =$  maximum pencils to be manuf.

$Z_1 = 1300 =$  maximum profit possible .

### Example 2.

**X Ltd. Produces two products  $p_1, p_2$  having profit of Rs. 4 \$ Rs. 3 each  $p_1, p_2$  require 4 hrs. & 2 hrs. of machining respectively, the total available machining time is 10 hours.  $P_1, p_2$  consume 2 units &  $8/3$  units of raw material respectively subject to a total of maximum 8 units. Any no. of  $p_2$  can be produced & sold but the no. of  $p_1$  must not be more than 6.**

**Formulate as a (P model & solve by the simplex method.)**

### Solution

$$\text{Maximise} = 4x_1 + 3x_2 + 0s_2 + 0s_3$$

Subject to :

$$\left. \begin{aligned} 4x_1 + 2x_2 + s_1 + 0s_2 + 0s_3 &= 10 \\ 2x_1 + \frac{8}{3}x_2 + 0s_1 + 0s_2 + 0s_3 &= 8 \\ x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_3 &= 6 \end{aligned} \right\} x_1, x_2 \geq 0$$

### Ist Tableau

$C_j$ (Rs.) (Rs.)	Basic Act.	Qty.	4 $x_1$	3 $x_2$	0 $S_1$	0 $S_2$	0 $S_3$
0	$S_1$	10	4	2	1	0	0
0	$S_2$	8	2	$8/3$	0	1	0
0	$S_3$	6	1	0	0	0	1
	$Z_j$ (Rs.)	0	0	0	0	0	0
	$C_j - Z_j$ (Rs.)		4	3	0	0	0

Pivot Column =  $x_1$

$$\text{Pivot Row} = S_1 \quad S_1 \left[ S_1 : \frac{10}{4} = \frac{5}{4}, S_2 : \frac{8}{2} = 4, S_3 : \frac{6}{1} = 6 \right]$$

Pivot Element = 4

$$\text{Updating } S_2 \text{ row: } \frac{10}{4} = 2.5, \frac{4}{4} = 1, \frac{2}{4} = 0.5, \frac{1}{4} = 0.25, 0, 0$$

**Up-dating  $S_2$**

$$8 - \left( 2x \frac{5}{2} \right) = 3$$

$$2 - (2x1) = 0$$

$$\frac{8}{3} - \left( 2x \frac{1}{2} \right) = \frac{5}{3}$$

$$0 - \left( 2x \frac{1}{4} \right) = -\frac{1}{2}$$

$$1 - (2x0) = 1$$

$$0 - (2x0) = 0$$

**row Up-dating  $S_3$  row**

$$8 - \left( 2x \frac{5}{2} \right) = \frac{7}{2}$$

$$1 - (1x1) = 0$$

$$0 - \left( 2x \frac{1}{2} \right) = \frac{1}{2}$$

$$0 - \left( 1x \frac{1}{4} \right) = -\frac{1}{4}$$

$$0 - (1x0) = 0$$

$$1 - (1x0) = 1$$

**Up-dating  $Z_j$  row**

$$Qty. - 4x \frac{5}{2} = 10$$

$$x_1 = 4$$

$$x_2 = 4x \frac{1}{2} = 2$$

$$S_1 = 4x \frac{1}{4} = 2$$

$$S_2 = 0$$

$$S_3 = 0$$

**Final Tableau**

C <sub>j</sub> (Rs.) (Rs.)	Basic Act.	Qty.	4 x <sub>1</sub>	3 x <sub>2</sub>	0 S <sub>1</sub>	0 S <sub>2</sub>	0 S <sub>3</sub>
4	x <sub>1</sub>	5/2	1	1/2	1/4	0	0
0	S <sub>2</sub>	3	0	5/3	-1/2	1	0
0	S <sub>3</sub>	7/2	0	-1/2	-1/4	1	1
	Z <sub>j</sub>	10	4	2	1	1	0
	C <sub>j</sub> -Z <sub>j</sub>		0	1	-1	1	0

$$\text{Pivot row} = S_2 \times \left[ x_1 + \frac{5}{2} \div \frac{1}{2} = 5, S_2 : 3 \div \frac{5}{3} = \frac{9}{5}, S_3 : \frac{7}{2} \div \frac{1}{2} = -7 \right]$$

$$\frac{9}{5} \rightarrow \text{Smallest \&}$$

$$\frac{7}{2} \rightarrow \text{This will not be considered Read again Pivot row selection rule.}$$

Pivot element 5/3

**Up dating pivot row:**

$$\frac{3}{5} = \frac{9}{5}, 0, 1, \frac{-1}{5} = \frac{-3}{10}, \frac{1}{5} = \frac{3}{5} = 0$$

**Up –dating  $x_1$  row:**

$$\frac{5}{2} - \left( \frac{1}{2} \times \frac{9}{5} \right) = \frac{8}{5}$$

$$1 - \left( \frac{1}{2} \times 0 \right) = 1$$

$$\frac{1}{2} - \left( \frac{1}{2} \times 1 \right) = 0$$

$$\frac{1}{4} - \left( \frac{1}{2} \times \frac{1}{5} \right) = \frac{2}{5}$$

$$0 - \left( \frac{1}{2} \times \frac{3}{5} \right) = \frac{-3}{10}$$

$$0 - \left( \frac{1}{2} \times 0 \right) = 0$$

**Updating  $S_3$ :**

$$\frac{7}{2} - \left( -\frac{1}{2} \times \frac{9}{5} \right) = \frac{22}{5}$$

$$0 - \left( -\frac{1}{2} \times 0 \right) = 0$$

$$-\frac{1}{2} - \left( -\frac{1}{2} \times 1 \right) = 0$$

$$-\frac{1}{4} - \left( -\frac{1}{2} \times \frac{-3}{4} \right) = \frac{-2}{5}$$

$$0 - \left( -\frac{1}{2} \times \frac{3}{5} \right) = \frac{3}{10}$$

$$1 - \left( -\frac{1}{2} \times 0 \right) = 1$$

**Up-dating  $Z_j$  row:**

$$Q + y = 4 \times \frac{8}{5} + \frac{3 \times 9}{5}$$

### IIIrd Tableau

$C_j$ (Rs.)			3	4	0	0	0
(Rs.)	Basic	Qty.	$X_1$	$X_2$	$S_1$	$S_2$	$S_3$
	Act.						
0	$X_1$	$8/5$	1	0	$2/5$	$-3/10$	0
3	$X_2$	$9/5$	0	1	$-3/10$	$3/5$	0
4	$S_3$	$22/5$	0	0	$-2/5$	$3/10$	1
	$Z_j$	$59/5$	4	3	$7/10$	$3/5$	0
	$C_j - Z_j$		0	0	$-7/10$	$-3/5$	0

There are no +ve values in  $C_j - Z_j$  row, optimal soln. Is reached.

Hence  $X_1 = 8/5$ ,  $X_2 = 9/5$  & max. =  $59/5$  Ans.

Dear students, we have now reached the end of our discussion scheduled for today.

See you all in the next lecture.

Bye.

