Unit 1

Linear Programming

Lesson 2: Introduction to linear programming
And
Problem formulation

Definition And Characteristics Of Linear Programming

Linear Programming is that branch of mathematical programming which is designed to solve optimization problems where all the constraints as will as the objectives are expressed as Linear function .It was developed by George B. Denting in 1947. Its earlier application was solely related to the activities of the second' World War. However soon its importance was recognized and it came to occupy a prominent place in the industry and trade.

Linear Programming is a technique for making decisions under certainty i.e.; when all the courses of options available to an organisation are known & the objective of the firm along with its constraints are quantified. That course of action is chosen out of all possible alternatives which yields the optimal results. Linear Programming can also be used as a verification and checking mechanism to ascertain the accuracy and the reliability of the decisions which are taken solely on the basis of manager's experience-without the aid of a mathematical model.

Some of the definitions of Linear Programming are as follows:

"Linear Programming is a method of planning and operation involved in the construction of a model of a real-life situation having the following elements:

(a) Variables which denote the available choices and

(b) the related mathematical expressions which relate the variables to the controlling conditions, reflect clearly the criteria to be employed for measuring the benefits flowing out of each course of action and providing an accurate measurement of the organization's objective. The method maybe so devised' as to ensure the selection of the best alternative out of a large number of alternative available to the organization

Linear Programming is the analysis of problems in which a Linear function of a number of variables is to be optimized (maximized or minimized) when whose variables are subject to a number of constraints in the mathematical near inequalities.

From the above definitions, it is clear that:

(i) Linear Programming is an optimization technique, where the underlying objective is either to maximize the profits or to minim is the Cosp.

(ii) It deals with the problem of allocation of finite limited resources amongst different competiting activities in the most optimal manner.

(iil) It generates solutions based on the feature and characteristics of the actual problem or situation. Hence the scope of linear programming is very wide as it finds application in such diverse fields as marketing, production, finance & personnel etc.

(iv) Linear Programming has be-en highly successful in solving the following types of problems :

(a) Product-mix problems

(b) Investment planning problems

(c) Blending strategy formulations and

(d) Marketing & Distribution management.

(v) Even though Linear Programming has wide & diverse' applications, yet all LP problems have the following properties in common:

*(a)*The objective is always the same (i.e.; profit maximization or cost minimization).

(b) Presence of constraints which limit the extent to which the objective can be pursued/achieved.

(c) Availability of alternatives i.e.; different courses of action to choose from, and

(d) The objectives and constraints can be expressed in the form of linear relation.

(VI) Regardless of the size or complexity, all LP problems take the same form i.e. allocating scarce resources among various compete ting alternatives. Irrespective of the manner in which one defines Linear Programming, a problem must have certain basic characteristics before this technique can be utilized to find the optimal values.

The characteristics or the basic assumptions of linear programming are as follows:

1. Decision or Activity Variables & Their Inter-Relationship. The decision or activity variables refer to any activity which are in competition with other variables for limited resources. Examples of such activity variables are: services, projects, products etc. These variables are most often inter-related in terms of utilization of the scarce resources and need simultaneous solutions. It is important to ensure that the relationship between these variables be linear.

2. Finite Objective Functions. A Linear Programming problem requires a clearly defined, unambiguous objective function which is to be optimized. It should be capable of being expressed as a liner function of the decision variables. The single-objective optimization is one of the most important prerequisites of linear programming. Examples of such objectives can be: cost-minimization, sales, profits or revenue maximization & the idle-time minimization etc

3. Limited Factors/Constraints. These are the different kinds of limitations on the available resources e.g. important resources like availability of machines, number of man hours available, production capacity and number of available markets or consumers for finished goods are often limited even for a big organisation. Hence, it is rightly said that each and every organisation function within overall constraints 0 11 both internal and external.

These limiting factors must be capable *of* being expressed as linear equations or in equations in terms of decision variables

4. Presence of Different Alternatives. Different courses *of* action or alternatives should be available to a decision maker, who is required *to* make the decision which is the most effective or the optimal.

For example, many grades *of* raw material may be available, the' same raw material can be purchased from different supplier, the finished goods can be sold to various markets, production can be done with the help *of* different machines.

5. Non-Negative Restrictions. Since the negative values *of* (any) physical quantity has no meaning, therefore all the variables must assume non-negative values. If some of the variables is unrestricted in sign, the non-negativity restriction can be enforced by the help *of* certain mathematical tools – without altering the original informati9n contained in the problem.

6. Linearity Criterion. The relationship among the various decision variables must be directly proportional Le.; Both the objective and the constraint,$ must be expressed in terms of linear equations or inequalities. For example. if--6ne *of* the factor inputs (resources like material, labour, plant capacity etc.) Creases, then it should result in a proportionate manner in the final output. These linear equations and in equations can graphically be presented as a straight line.

7. Additively. It is assumed that the total profitability and the total amount *of* each resource utilized would be exactly equal to the sum *of* the respective individual amounts. Thus the function or the activities must be additive - and the interaction among the activities *of* the resources does not exist.

8. Mutually Exclusive Criterion. All decision parameters and the variables are assumed to be mutually exclusive In other words, the occurrence of anyone variable rules out the simultaneous occurrence of other such variables.

9. Divisibility. Variables may be assigned fractional values. i.e.; they need not necessarily always be in whole numbers. If a fraction *of* a product can not be produced, an integer programming problem exists.

Thus, the continuous values of the decision variables and resources must be permissible in obtaining an optimal solution.

10. Certainty. It' is assumed that conditions *of* certainty exist i.e.; all the relevant parameters or coefficients in the Linear Programming model are ful1y and completely known and that they don't change during the period. However, such an assumption may not hold good at all times.

11. Finiteness. 'Linear Programming assumes the presence *of* a finite number *of* activities and constraints without which it is not possible to obtain the best or the optimal solution.

Advantages & Limitations Of Linear Programming

Advantages of Linear Programming .Following are some *of* the advantages of Linear Programming approach:

1. Scientific Approach to Problem Solving. Linear Programming is the application *of* scientific approach to problem solving. Hence it results in a better and true picture *of* the problems-which can then be minutely analysed and solutions ascertained.

2. Evaluation of All Possible Alternatives. Most *of* the problems faced by the present organisations are highly complicated - which can not be solved by the traditional approach to decision making. The technique *of* Linear Programming ensures that'll possible solutions are generated - out of which the optimal solution can be selected.

3. Helps in Re-Evaluation. Linear Programming can also be used in .re-evaluation *of* a basic plan for changing conditions. Should the conditions change while the plan is carried out only partially, these conditions can be accurately determined with the help *of* Linear Programming so as to adjust the remainder *of* the plan for best results.

4. Quality of Decision. Linear Programming provides practical and better quality *of* decisions' that reflect very precisely the limitations *of* the system i.e.; the various restrictions under which the system must operate for the solution *to* be optimal. If it becomes necessary to deviate from the optimal path, Linear Programming can quite easily evaluate the associated costs or penalty.

5. Focus on Grey-Areas. Highlighting *of* grey areas or bottlenecks in the production process is the *most* significant merit *of* Linear Programming. During the periods *of* bottlenecks, imbalances occur in the production department. Some *of* the machines remain idle *for* long periods *of* time, while the other machines are unable toffee the demand even at the peak performance level.

6. Flexibility. Linear Programming is an adaptive & flexible mathematical technique and hence can be utilized in analyzing a variety of multi-dimensional problems quite successfully.

7. Creation of Information Base. By evaluating the various possible alternatives in the light *of t*he prevailing constraints, Linear Programming models provide an important database from which the allocation *of* precious resources can be don rationally and judiciously.

8. Maximum optimal Utilization of Factors of Production. Linear Programming helps in optimal utilization *of* various existing factors of production such as installed capacity,. labour and raw materials etc.

Limitations of Linear Programming.

Although Linear Programming is a highly successful having wide applications in business and trade for solving optimization' problems, yet it has certain demerits or defects.

Some *of* the important-limitations in the application *of* Linear Programming are as follows:

1. Linear Relationship. Linear Programming models can be successfully applied only in those situations where a given problem can clearly be represented in the form *of* linear relationship between different decision variables. Hence it is based on the implicit assumption that the objective as well as all the constraints or the limiting factors can be stated in term of linear expressions - which may not always hold *good* in real life situations. In practical business problems, many objective function & constraints can not be expressed linearly. Most of \he business problems can be expressed quite easily in the form of a quadratic equation (having a power 2) rather than in the terms of linear equation. Linear Programming fails to operate and provide optimal solutions in all such cases.

e.g. A problem capable of being expressed in the form of:

$ax^2+bx+c = 0$ where $a \# 0$ can not be solved with the help of Linear Programming techniques.

2. Constant Value of objective & Constraint Equations. Before a Linear Programming technique could be applied to a given situation, the values or the coefficients of the objective function as well as the constraint equations must be completely known. Further, Linear Programming assumes these values to be constant over a period of time. In other words, if the values were to change during the period of study, the technique of LP would loose its effectiveness and may fail to provide optimal solutions to the problem.

However, in real life practical situations often it is not possible to determine the coefficients of objective function and the constraints equations with absolute certainty. These variables in fact may, lie on a probability distribution curve and hence at best, only the Iikelil1ood of their occurrence can be predicted. Mover over, often the value's change due to extremely as well as internal factors during the period of study. Due to this, the actual applicability of Linear
Programming tools may be restricted.

3. No Scope for Fractional Value Solutions. There is absolutely no certainty that the solution to a LP problem can always be quantified as an integer quite often, Linear Programming may give fractional-varied answers, which are rounded off to the next integer. Hence, the solution would not be the optimal one. For example, in finding out 'the pamper of men and machines required to perform a particular job, a fractional Larson-integer solution would
be meaningless.

4. Degree Complexity. Many large-scale real life practical problems can not be solved by employing Linear Programming techniques even with the help of a computer due to highly complex and Lengthy calculations. Assumptions and approximations are required to be made so that $e, given problem can be broken down into several smaller problems and, then solve separately. Hence, the validity of the final result, in all such cases, may be doubtful:

5. Multiplicity of Goals. The long-term objectives of an organisation are not confined to a single goal. An organisation ,at any point of time in its operations has a multiplicity of goals or the goals hierarchy - all of which must be attained on a priority wise basis for its long term growth. Some of the common goals can be Profit maximization or *cost* minimization, retaining market share, maintaining leadership position and providing quality service to the consumers. In cases where the management has conflicting, multiple goals, Linear Programming model fails to provide an optimal solution. The reason being that under Linear Programming techniques, there is only one goal which can be expressed in the objective function. Hence in such circumstances, the situation or the given problem has to be solved by the help of a different mathematical programming technique called the *"Goal Programming"*.

6. Flexibility. Once a problem has been properly quantified in terms of objective function and the constraint equations and the tools of Linear Programming are applied to it, it becomes very difficult to incorporate any changes in the system arising on account of any change in the decision parameter. Hence, it lacks the desired operational flexibility.

Mathematical model of LPP.

Linear Programming is a mathematical technique for generating & selecting the optimal or the best solution for a given objective function. Technically, Linear Programming may be formally defined as a method of optimizing (i.e.; maximizing or minimizing) a linear function for a number of constraints stated in the form of linear in equations.

Mathematically the problem of Linear Programming may be stated as that of the optimization of linear objective function of the following form :

$$Z = C_1 x_1 + c_2 x_2 + ...C_i x_i + ... + C_n X_n$$

Subject to the Linear constrains of the form:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_{3.} .... + a_{1i} x_i + ... + a_{\ln} x_n \overset{\leq}{\geq} b_1$$

$$a_{j1} x_1 + a_{22} x_2 + a_{23} x_{3.} .... + a_{2i} x_i + ... + a_{2n} x_n \overset{\leq}{\geq} b_2$$

$$a_{j1} x_1 + a_{22} x_2 + a_{33} x_{3.} .... + am_i x_i + ... + a_{jn} x_n \overset{\leq}{\geq} bm_1$$

$$\leq$$

$$am_1 n_1 + am_2 n_2 + am_3 x_3 + ...am_i x_i ....a_{mn} X_n \leq b_m$$

These are called the non-negative constraints. From the above, it is linear that a LP problem has:

(I) linear objective function which is to be maximized or minimized.

(ii) *var*ious linear constraints which are simply the algebraic statement of the limits of the resources or inputs at the disposal.

(iii) *No*n-negatively constraints.

Linear Programming is one of the few mathematical tools that can be used to provide solution to a wide variety of large, complex managerial problems.

For example, an oil refinery can vary its product-mix by its choice among the different grades of crude oil available from various parts of the world. Also important is the process selected since parameters such as temperature would also affect the yield. As prices and demands vary, a Linear Programming model recommends which inputs and processes to use in order to maximize the profits.

Livestock gain in value as they grow, but the rate of gain depends partially on the feed choice of the proper combination of ingredients to maximize the net gain. This value can be expressed in terms of Linear Programming. A firm which distributes products over a large territory faces an unimaginable number of different choices in deciding how best to meet demand from its network of godown and warehouses. Each warehouse has a very limited number of items and demands often can not be met from the nearest warehouse. If their are 25 warehouses and 1,000 customers, there are 25,000 possible match ups between customer and warehouse. LP can quickly recommend the shipping quantities and destinations so as to minimize the cost of total distribution.

These are just a few of the managerial problems that have been addressed successfully by LP. A few others are described throughout this text. Project scheduling can be improved by allocating funds appropriately among the most critical task so as to most effectively reduce the overall project duration. Production planning over a year or more can reduce costs by careful timing of the use of over time and inventory to control changes in the size of the workforce. In the short run, personnel work schedules must take into consideration not only the production, work preferences for day offs and absenteeism etc.

Besides recommending solutions to problems like these, LP can provide useful information for managerial decisions, that can be solved by Linear Programming. The application, however, rests on certain postulates and assumptions which have to hold good for the optimality of the solution to be effective during the planning period.

Applications Of Linear Programming Techniques In Indian Economy

In a third world developing country like India, the various factors of productions such as skilled labour, capital and raw material etc. are very precious and scarce. The policy planner is, therefore faced with the problem of scarce resource allocation to meet the various competing demands and numerous conflicting objectives. The traditional and conventional methods can no longer be applied in the changed circumstances for solving this problem and are hence fast losing their importance in the current economy. Hence, the planners in our country are continuously and constantly in search of highly objective and result oriented techniques for sound and proper decision making which can be effective at all levels of economic planning. Nonprogrammed decisions consist of capacity expansion, plant location, product line diversification, expansion, renovation and modernization etc. On the other hand, the programmed decisions consist of budgeting, replacement, procurement, transportation and maintenance etc.

In These modern times, a number of new and better methods ,techniques and tools have been developed by the economists all over the globe. All these findings form the basis of operations research. Some of these well-known operations research techniques have been successfully applied in Indian situations, such as: business forecasting, inventory models - deterministic and probabilistic, Linear Programming.Goal programming, integer programming and dynamic programming etc.

The main applications of the Linear Programming techniques, in Indian context are as follows:

1. Plan Formulation. In the formulation of the country's five year plans, the Linear Programming approach and econometric models are being used in various diverse areas such as : food grain storage planning, transportation, multi-level planning at the national, state and district levels and urban systems.

2. Railways. Indian Railways, the largest employer in public sector undertakings, has successfully applied the methodology of Linear Programming in various key areas.

For example, the location of Rajendra Bridge over the Ganges linking South Bihar and North Bihar in Mokama in preference to other sites has been achieved only by the help of Linear Programming.

3. Agriculture Sector. Linear Programming approach is being extensively used in agriculture also. It has been tried on a limited scale for the crop rotation mix of cash crops, food crops and to/ascertain the optimal fertilizer mix.

4. Aviation Industry. Our national airlines are also using Linear Programming in the selection of routes and allocation of air-crafts to various chosen routes. This has been made possible by the application of computer system located at the headquarters. Linear Programming has proved to be a very useful tool in solving such problems. '

5. Commercial Institutions. The commercial institutions as well as the individual traders are also using Linear Programming techniques for cost reduction and profit maximization. The oil refineries are using this technique for making effective and optimal blending or mixing decisions and for the improvement of finished products.

6. Process Industries. Various process industries such as paint industry makes decisions pertaining to the selection of the product mix and locations of warehouse for distribution etc. with the help of Linear Programming techniques. This mathematical technique is being extensively used by highly reputed corporations such as TELCO for deciding what castings and forging to be manufactured in own plants and what should be purchased from outside suppliers. '

7. Steel Industry. The major steel plants are using Linear Programming techniques for determining the optimal combination of the final products such as : billets, rounds, bars, plates and sheets.

8. Corporate Houses. Big corporate houses such as Hindustan Lever employ these techniques for the distribution of consumer goods throughout the country. Linear Programming approach is also used for capital budgeting decisions such as the selection of one project from a number of different projects.

Main Application Areas Of Linear Programming

In the last few decades since 1960s, no other mathematical tool or technique has had such a profound impact on the management's decision making criterion as Linear Programming well and truly it is one of the most important decision making tools of the last century which has transformed the way in which decisions are made and businesses are conducted. Starting with the Second World War till the Y -2K problem in computer applications, it has covered a great distance.

We discuss below some of the important application areas of Linear Programming:

I. Military Applications. Paradoxically the most appropriate example of an organization is the military and worldwide, Second World War is considered to be one of the best managed or organized events in the history of the mankind. Linear Programming is extensively used in military operations. Such applications include the problem of selecting an air weapon system against the enemy so as to keep them pinned down and at the same time minimizes the amount of fuel used. Other examples are dropping of bombs on pre-identified targets from aircraft and military assaults against localized terrorist outfits.

2. Agriculture. Agriculture applications fall into two broad categories, farm economics and farm management. The former deals with the agricultural economy of a nation or a region, while the latter is concerned with the problems of the individual form. Linear Programming can be gainfully utilized for agricultural planning e:g. allocating scarce limited resources such as capital, factors of production like labour, raw material etc. in such a way 'so as to maximize the net revenue.

3. Environmental Protection. Linear programming is used to evaluate the various possible a1temative for handling wastes and hazardous materials so as to satisfy the stringent provisions laid down by the countries for environmental protection. This technique also finds applications in the analysis of alternative sources of energy, paper recycling and air cleaner designs.

4. Facilities Location. Facilities location refers to the location nonpublic health care facilities (hospitals, primary health centers) and' public recreational facilities (parks, community hal1s) and other important facilities pertaining to infrastructure such as telecommunication booths etc. The analysis of facilities location can easily be done with the help of Linear Programming.

Apart from these applications, LP can also be used to plan for public expenditure and drug control. '

5. Product-Mix. The product-mix of a company is the existence of various products that the company can produce and sell. However, each product in the mix requires finite amount of limited resources. Hence it is vital to determine accurately the quantity of each product to be produced knowing their profit margins and the inputs required for producing them. The primary objective is to maximize the profits of the firm subject to the limiting factors within which it has to operate.

6. Production. A manufacturing company is quite often faced with the situation where it can manufacture several products (in different quantities) with the use of several different machines. The problem in such a situation is to decide which course of action will maximize output and minimize the costs.

Another application area of Linear Programming in production is the assembly by-line balancing - where a component or an item can be manufactured by assembling different parts. In such situations, the objective of a Linear Programming model is to set the assembly process in the optimal (best possible) sequence so that the total elapsed time could be minimized.

7. Mixing or Blending. Such problems arise when the same product can be produced with the help of a different variety of available raw-materials each having a fixed composition and cost. Here the objective is to determine the minimum cost blend or mix (Le.; the cost minimizations) and the various constraints that operate are the availability of raw materials and restrictions on some of the product constituents.

8. Transportation & Trans-Shipment. Linear Programming models are employed to determine the optimal distribution system i.e.; the best possible channels of distribution available to an organisation for its finished product sat minimum total cost of transportation or shipping from company's godown to the respective markets. Sometimes the products are not transported as finished products but are required to be manufactured at various. sources. In such a situation, Linear Programming helps in ascertaining the minimum cost of producing or manufacturing at the source and shipping it from there.

9. Portfolio Selection. Selection of desired and specific investments out of a large number of investment' options available10 the managers (in the form of financial institutions such as banks, non-financial institutions such as mutual funds, insurance companies and investment services etc.) is a very difficult task, since it requires careful evaluation of all the existing options before arriving at C\ decision. The objective of Linear Programming, in such cases, is to find out the allocation/which maximizes the total expected return or minimizes the total risk under different situations.

10. Profit Planning & Contract. Linear Programming is also quite useful in profit planning and control. The objective is to maximize the profit margin from investment in the plant facilities and machinery, cash on hand and stocking-hand.

11. Traveling Salesmen Problem. Traveling salesmen problem refers to the problem of a salesman to find the shortest route originating from a particular city, visiting each of the specified cities and then returning back to the originating point of departure. The restriction being that no *city* must be visited more than once during a particular tour. Such types of problems can quite easily be solved with the help of Linear Programming.

12. Media Selection/Evaluation. Media selection means the selection of the optimal media-mix so as to maximise the effective exposure. The various constraints in this case are: Budget limitation, different rates for different media (i.e.; print media, electronic media like radio and T.V. etc.) and the minimum number of repeated advertisements (runs) in the various media. The use of Linear Programming facilities like the decision making process.

13. Staffing. Staffing or the man-power costs are substantial for a typical organisation which make its products or services very costly. Linear Programming techniques help in allocating the optimum employees (man-power or man-hours) to the job at hand. The overall objective is to minimize the total man-power or overtime costs.

14. Job Analysis. Linear Programming is frequently used for evaluation of jobs in an organisation and also for matching the right job with the right worker.

15. Wages and Salary Administration. Determination of equitable salaries and various incentives and perks becomes easier with the application of Linear Programming. LP tools" can also be utilized to provide optimal solutions in other areas of personnel management such as training and development and recruitment etc.

Linear Programming problem Formation

Steps In Formulating A Linear Programming Model

Linear programming is one of the most useful techniques for effective decision making. It is an optimization approach with an emphasis on providing the optimal solution for resource allocation. How best to allocate the scarce organisational or national resources among different competing and conflicting needs (or uses) forms the core of its working. The scope for application of linear programming is very wide and it occupies a central place in many diversified decisional problems. The effective use and application of linear programming requires the formulation of a realistic model which represents accurately the objectives of the decision making subject to the constraints in which it is required to be made.

The basic steps in formulating a linear programming model are as follows:

Step I. Identification of the decision variables. The decision variables (parameters) having a bearing on the decision at hand shall first be identified, and then expressed or determined in the form of linear algebraic functions or in equations.

Step II. Identification of the constraints. All the constraints in the given problem which restrict the operation of a firm at a given point of time must be identified in this stage. Further these constraints should be broken down as linear functions in terms of the pre-defined decision variables.

Step III. Identification of the objective. In the last stage, the objective which is required to be optimized (i.e., maximized or memorized) must be dearly identified and expressed in terms of the pre-defined decision variables.

Example 1

High Quality furniture Ltd. Manufactures two products, tables & chairs. Both the products have to be processed through two machines Ml & M2 the total machine-hours available are: 200 hours ofM1 and 400 hours of M2 respectively. Time in hours required for producing a chair and a table on both the machines is as follows**:**

Time in Hours

Machine	Table	Chair
MJ	7	4
M2	5	, 5

Profit from the Sale of table is Rs. 40 and that from a chair is Rs. 30 determine optimal mix of tables & chairs so as to maximized the total profit

Contribution.

Let x_1 = no. of tables produced and
X_2 = no. of Chairs produced

Step I. The objective function for maximizing the profit is given by maximize
$Z=50x_1 +30x_2$ (objective function)
(Since profit per unit from a table and a chair is Rs. 50 & Rs. 30 respectively).

Step II. List down all the constraints.

(i) Total time on machine M_1 can not exceed 200 hours.

$\therefore 7x1 + 4x2 \leq 200$

(Since it takes 7 hours to produce a table & 4 hours to produce a chair on machine M_1)

(ii) Total time on machine M_2 cannot exceed 400 hours.

$\therefore 7x1 + 4x2 \leq 200$

(Since it takes 5 hours to produce both a table & a chair on machine M_2)

Step III Presenting the problem. The given problem can now be formulated as a linear programming model as follows:

Maximise $Z = 50x_1 + 30x_2$

Subject: : $7x_1 + 4x_2 \leq 200$

$5x_1 + 5x_2 \leq 400$

Further; $x1 + x_2 \geq 0$

(Since if x1 & x2 < 0 it means that negative quantities of products are being manufactured – which has no meaning).

Example 2.

Alpha Limited produces & sells 2 different products under the brand name black & white. The profit per unit on these products in Rs. 50 & Rs. 40 respectively. Both black & white employ the same manufacturing process which has a fixed total capacity of 50,000 man-hours. As per the estimates of the marketing research department of Alpha Limited, there is a market demand for maximum 8,000 units of Black & 10,000 units of white. Subject to the overall demand, the products can be sold in any possible combination. If it takes 3 hours to produce one unit of black & 2 hours to produce one unit of white, formulate the about as a linear programming model.

Let x_1, x_2 denote the number of units produced of products black & white respectively.

Step 1: The objective function for maximizing the profit is given by : maximize

$Z= 50x_1 + 40x_2$ (objective function)

Step II: List down all the constraints.

(i) Capacity or man-hours constraint:

(Since it takes 3 hours to produce one unit of x1 & hours to produce 1 unit of x2 & the total available man – hours are 50,000)

(ii) Marketing constraints:

$x_1 \leq 8,000$

(Since maximum 8,000 units of x_1 can be sold)

$x2 \leq 10,000$

(Since maximum 10,000 units of x_2 can be sold).

Step III: Presenting the problem. Now, the given problem can be written as a linear programming model in the follows:

Maximize $Z = 50x_1 + 50x_2$

Subject: : $3x_1 + 2x_2 \leq 50,000$

$x_1 \leq 8000$

$x_2 \leq 10,000$

Further; $x1 + x_2 \geq 0$

(Since if $x_1, x_2 < 0$, it means that negative Quantities of products are being manufactured – which has no meaning)

Example 3.

Good results company manufactures & sells in the export market three different kinds of products P_1 , P_2 & P_3. The anticipated sales for the three products are 100 units of P_1, 200 units of P_2 & 300 units of P_3. As per the terms of the contract Good results must produce at least 50 units of P_1 & 70 units of P_3. Following is the break – up of the various production times:

Product	**Production Hours per Unit**				**(Rs.) Unit Profit**
	Department (A)	**Department (B)**	**Department (C)**	**Department (D)**	
P_1	0.05	0.06	0.07	0.08	15
P_2	0.10	0.12	--	0.30	20
P_3	0.20	0.09	0.07	0.08	25
Available hours	**40.00**	**45.00**	**50.00**	**55.00**	

Management is free to establish the production schedule subject to the above constraints.

Formulate as a linear programming model assuming profit maximization criterion for Good results company.

Ans. Let X_1 , X_2 , X_3 denote the desired quantities of products P_1 , P_2 & P_3 respectively.

Step I. The objective function for maximizing total profits is given by:
Maximize Z $15x_1 + 20x_2 + 25x_3$ (objective function)
Step II. List down all the constraints.
The available production hours for each of the products must satisfy the following criterion for each department:

(i) $0.05x_1+0.10x_2+0.20x_3 \leq 40.00$
Total hours available on product – wise basis in Department A
(ii) $0.06x_1+0.12x_2+00.09x_3 \leq 45.00$
Total hours available on product – wise basis in Department B
(iii) $0.07x_1+0x_2+0.07x_3 \leq 50.00$
Total hours available on product – wise basis in Department C
(iv) $0.08x_1+0.30x_2+0.08x_3 \leq 55.00$
Total hours available on product – wise basis in Department D
(v) $50 \leq x_1 \leq 100$
Minimum 50 units of P_1 must be produced subject of maximum of 100 units.
(vi) $0 \leq x_2 \leq 200$
Maximum units of P_2 that can be sold is 200 units.
(vii) $70 \leq x_3 \leq 300$
Minimum 70 units of P_3 must be produced subject to a maximum of 300 units
(viii) Further , $x_1,x_2,x_3 \geq 0$
Since negative values of P_1 , P_2 & P_3 has no meaning
Step III: Presenting the Problem
The given problem ca be reduced as a LP model as under:
Maximise $Z=15x_1 + 20x_2 + 25x_3$
Subject to: $0.05x_1+0.10x_2+0.20x_3 \leq 40.00$
$0.06x_1+0.12x_2+00.09x_3 \leq 45.00$
$0.07x_1+0x_2+0.07x_3 \leq 50.00$
$0.08x_1+0.30x_2+0.08x_3 \leq 55.00$
$50 \leq x_1 \leq 100$
$0 \leq x_2 \leq 200$
$70 \leq x_3 \leq 300$
$x_1, x_2, x_3 \geq 0$

Example 4.
The management of Surya Chemicals is considering the optimal mix of two possible processes. The values of input & output for both these process are given as follows:

Process	Units – inputs		Units – Outputs	
	I_1	**I_2**	**O_1**	**O_2**
X	2	6	3	7
Y	4	8	5	9

Maximum 500 units of Input I_1 and 300 units of I_2 are available to Surya Chemicals in the local market. The forecasted demand for outputs O_I & O_2 are at least 5,000 units & 7,000 units respectively. The respective profits from process X & Y are Rs. 1,000 & Rs. 2,000 – per production run. You are required to formulate the above as a linear programming model.

Ans. Let X_1, X_2 represent the number of production runs of process x & y respectively .

Step I. The objective function for maximizing the total profits from both the process is given by:

Maximise Z $1000x_1 + 2000x_2$

Step II. List down all the constraints

(i) $2x_1 + 4x_2 \leq 500$

Maximum amount of input I_1 available for process x & y is 500 units

(ii) $6x_1 + 8x_2 \leq 300$

Maximum amount of input I_2 available for process x & y 300 units

(iii) $3x_1 + 5x_2 \leq 5{,}000$

Since market requirement is to produce at least 5000 units of O_1

(iv) $7x_1 + 9x_2 \leq 7{,}000$

Since market requirement is to produce at least 7000 units of O_2

Further; $x_1, x_2 \geq 0$ as always.

Step III. Presenting the model.

Now, the current LP model can be presented as:

Maximise $Z = 1000x_1 + 2000\,x_2$

Subject to: $2x_1 + 4x_2 \leq 500$

$6x_1 + 8x_2 \leq 300$

$3x_1 + 5x_2 \leq 5{,}000$

$7x_1 + 9x_2 \leq 7{,}000$

$x_1, x_2 \geq 0$

Example 5. Chocolate India Ltd. Produces three varieties of Chocolates – Hard, mild & soft from three different inputs I_1, I_2 & I_3 one unit of Hard requires 2 units of I_1 and 4 unit of I_2. One unit of mild requires 5 units of I_1, 4 units of I_2 and 3 units of I_3 and one unit of soft requires 10 units of I_1 & 15 units of I_3. The total available of inputs in the company's warehouse is as under:

I_1	-	100 units
I_2	-	400 units
I_3	-	50 units

The profit per unit for hard, mild & soft are Rs. 20, Rs. 30 and Rs. 40 respectively.

Formulate the problem so as to maximize the total profit by using linear programming.

At the beginning, it is better to present the problem in a tabular form

Product	Inputs Required			Profit (Rs. Per unit)
	I_1	I_2	I_3	
Hard	2	4	--	20
Mild	5	4	3	30
Soft	10	--	15	40
Maximum availability of inputs	100	400	50	

Let x_1, x_2 & x_3 denote the no. of units of the three varies of chocolate – Hard, mild & soft respectively.

Step 1 The objective function for maximizing the profit is:

Maximise $Z = 20x_1 + 30x_2 + 40x_3$ (objective function)

Step II List down all the constraints:

(i) $2x_1 + 5x_2 + 10x_3 \leq 100$

(Since Input I_1 required for the three products is 2, 5 & 10 units respectively subject to a maximum of 100 units).

(ii) $4x_1 + 4x_2 + 0x_3 \leq 400$

(Since input I_2 required for the three products is 4,4 & 0 units respectively subject to a maximum of 400 units).

(iii) $0x_1 + 3x_2 + 15x_3 \leq 50$

(Since input I_3 required for he three products is 0, 3 & 15 units respectively subject to a maximum of 50 units).

Step III. Presenting the problem. The given problem can now be formulated as a linear programming model as follows:

Maximise $Z = 20x_1 + 30x_2 + 40x_3$

Subject to:

$$2x_1 + 5x_2 + 10x_3 \leq 100$$
$$4x_1 + 4x_2 + 0x_3 \leq 400$$
$$0x_1 + 3x_2 + 15x_3 \leq 50$$

Further: $x_1, x_2, x_3 \geq 0$

Example 6 Safe & sound Investment Ltd. Wants to invest up to Rs. 10 lakhs into various bonds. The management is currently considering four bonds, the detail on return & maturity of which are as follows:

Bond	Type	Return	Maturity Time
α	Govt.	22%	15 years 5 years
β	Govt. Industrial	18% 28%	20 years 3 years
γ	Industrial	16%	

θ			

The company has decided not to put less than half of its investment in the government bonds and that the average age of the portfolio should not be more than 6 years. The investment should be such which maximizes the return on investment, subject to the above restriction.

Formulate the above as a LP problem.

Ans. Let

	X_1 = amount to be invested in bond	α	Govt.
	X_2 = amount to be invested in bond	β	Govt.
	X_3 = amount to be invested in bond	γ	Industrial
	X_4 = amount to be invested in bond	θ	Industrial

Step I. The objective function which maximizes the return on investment

Is: Maximise $Z = 0.22x_1 + 0.18x_2 + 0.28x_3 + 0.16x_4$

(Based on the respective rate of respective rate of return for each bond.

Step II. List down all the constraints:

(i) Sum of all the investments can not exceed the total fund available.

$\therefore x_1 + x_2 + x_3 + x_4 \leq 10{,}00{,}000$

(ii) Sum of investments in government bonds should not be less than 50%.

$\therefore x_1 + x_2 \leq 5{,}00{,}000$

(iii) Average : $\dfrac{15x_1 + 5x_2 + 20x_3 + 3x_4}{x_1 + x_2 + x_3 + x_4} \leq 6 \left(\dfrac{Numeratordenotes}{maturilyperiod} \right)$

Or $15x_1 + 5x_2 + 20x_3 + 3x_4 \leq 6x_1 + 6x_2 + 6x_3 + 6x_4$

Or $(15-6)x_1 + (5-6)x_2 (20-6)x_3 + (3-6)x_4 \leq 0$

$\therefore 9x_1 - x_2 + 14x_3 - 3x_4 \leq 0$

Step III. Presenting the problem

The given problem can now be formulated as a LP model:

Maximise $Z = 0.22x_1 + 0.18x_2 + 0.28x_3 + 0.16x_4$

Subject to: $x_1 + x_2 + x_3 + x_4 \leq 10{,}00{,}000$

$x_1 + x_2 \leq 5{,}00{,}000$

$9x_1 - x_2 + 14x_3 - 3x_4 \leq 0$

Further ; $x_1 + x_2 + x_3 + x_4 \leq 0$

Example7 Good products Ltd. Produces its product in two plants P1 & P2 and distributes this product from two warehouses W1 & W2. Each plant can produce a maximum of 80 units. Warehouse W1 can sell 100 units while W2 can sell only 60 units. Following table shows the cost of shipping one unit from plants to the warehouses:

To / from	(cost in Rs)	
	Warehouse (W_1)	**Warehouse (W_2)**
Plant (*P_1*)	**40**	**60**
Plant (*P_1*)	**70**	**75**

Determine the total no. of units to be shipped for each plant to each warehouse, such that the capacity of plants is not exceeded, demand at each warehouse is fully satisfied & the total cost of transportation is minimized.

Ans. Let S_{ij} = No. of units shipped form plant i to warehouse j.
Where i = p_1, p_2 and j = w_1, w_2

Further assume C to be the transportation cost of shipping a unit form plant to the warehouse then, we have:

C_{p1w1} = cost of shipping a unit form plant p_1 to warehouse $w_{1,}$

C_{p1w2} = cost of shipping a unit form plant p_1 to warehouse $w_{2,}$

C_{p2w1} = cost of shipping a unit form plant p_2 to warehouse w_1 &

C_{p2w2} = cost of shipping a unit form plant $p_1$2to warehouse $w_{2,}$
Formulate the problem in a tabular from as under.

	P_1 to w_1	P_1 to w_2	P_2 to w_1	P_2 to w_2	Resources
Warehouse W_1 demand	1	0	1	0	100 units
Warehouse w_2 demand	0	1	0	1	60 units
Plant P_2 supply	1	1	0	0	80 units
Plant p_2 supply	0	0	1	1	80 units
shipping costs (Rs.)	40	60	70	75	

Formulating as a LP Noble in the usual manner:

Objective – function

Minimize $Z = 40\ C_{P1W1} + 60\ C_{P1W2} + 70C_{P2W1} + 75\ C_{p2w2}$
Ware hose constraints.

$C_{p1w1} + C_{p1w2} = 100$

$C_{p1w2} + C_{p2w2} = 60$

Plant constraints

$C_{p1w1} + C_{p1w2} = 80$
$C_{p1w2} + C_{p2w2} = 80$
Where $C_{p1w1}, C_{p1w2}, C_{p2w1}, C_{p2w2} \leq 0$
(since these denote costs in a minimization problem)

Example 8. To maintain good health, a person must fulfil certain minimum daily requirements of several kinds of nutrients. For the sake of simplicity let us assume that only three kinds of these needs to be considered calcium, protein vitamin A. also assume that the person's diet is to consist of only 2 food items, I & II; whose prices & nutrient content's are given in the following table. Find out the optimal combination of the two food items that will satisfy the daily requirements & entail the least cost.

Food	Calcium	Protein	Vitamin A	Cost of unit
FI F2	10 4	5 5	2 6	6 1
Daily minimum Requirement	20	20	12	

Ans. Let x_1 = Quantity of F_1 to be purchased
X_2 = Quantity of F_2 to be purchased

Step 1. the objective function for minimizing the total cost is given by:
Minimize $Z = 6x_1 + x_2$ (objective – function)

Step II. List down all the constraints:
(i) calcium constraint: $10x_1 + 4x_2 \geq 20$

(ii) protein constraint: $5x_1 + 5x_2 \geq 20$
(iii) vitamin constraint: $2x_1 + 6x_2 \geq 12$

step III. Presenting as a Lp model:

minimize $Z = 6x_2 + X_2$

$10x_1 + 4x_2 \geq 20$
$5x_1 + 5x_2 \geq 20$
$2x_1 + 6x_2 \geq 12$

further $x_1, x_2 \geq 0$

Example9. A steel plant manufactures two grades of steel S_1 & S_2 . Data given below shows the total resources consumed & profit per unit associated with S_1 & S_2 .iron and labour are the only resources which are consumed in the manufacturing process. The manager of the firm wishes to determine the different units of S_1 & S_2. which should be manufactured to maximize the total profit.

Resource utilized	Unit-requirement		Amount
	S_1	S_2	Available
Lron (kg) labour (Hours)	30 5	20 10	300 110
Profit (Rs.)	6	8	

Ans. Let x_1 = no. of units of S_1 to be manufactured.
x_2 = no. of units of S_2 to be manufactured.

Step I The objective function for maximizing to be manufactured.

$Z = 6x_1 + 8x_2$ (objective – function)

(Since profit per unit of S_1 & S_2 are Rs. 6 and Rs. 8 respectively)

Step II. List down all the constraints:

(i) Iron – constraint:

$30x_1 + 20x_2 \leq 300$

(ii) Labour – constraint:

$5x_1 + 10x_2 \leq 110$

Step III. Presenting as a LP Model

Maximize $Z = 6x_1 + 8x_2$

Subject to: $30x_1 + 20x_2 \leq 300$

$5x_1 + 10x_2 \leq 110$

Further : $x_1, x_2 \geq 0$

Example 10. Mr. Khanna is exploring the various investment options for maximizing his return on investment. The investments that can be made by him pertain to the following areas:

Govt. bonds, fixed deposits of companies equity shares time deposits in banks, Indira Vikas Patra & real estate.

The data pertaining to the return on investment, the number of years for which the funds will be blocked to earn this return on investment and the related risks are as follows:

Option	**Return**	**No. of years**	**Risk factor**
Govt. bonds	**6%**	**15**	**1**
Company deposits	**15%**	**3**	**3**
Equity shares	**20%**	**6**	**7**

Time deposits	**10%**	**3**	**1**
Indira Vikas Patra	**12%**	**6**	**1**
Real Estate	**25%**	**10**	**2**

The average risk required by Mr. Khanna should not exceed 3 and the funds should not be fixed for more than 20 years. Further he would necessarily Formulate the above as a LP model.

Ans. Following the familiar methods of Linear programming, the objective function is:

Maximize $Z = 0.06x_1 + 0.15x_2 + 0.20x_3 + 0.10x_4 + 0.12x_5 + 0.25x_6$

(Maximizing total return on investment)

Subject to the following constraint equations:

$15x_1 + 3x_2 + 6x_3 + 3x_4 + 6x_5 + 10x_6 \le 20$

($\therefore$ *Funds should not be fixed for more than* 20 *years*)

$x_1 + 3x_2 + 7x_3 + x_4 + 2 \le 3$ ($\therefore$ Risk required at most = 3)

$x_6 \ge 0.40$ ($\therefore$ Investment in real estate is at least 40%)

Further $xi \ge 0 \; for \; i = 1,2....6$

i.e., $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$

where x_1 = Percentage of total funds to be invested in Govt. – bonds.

x_2 = Percentage of total funds to be invested in company deposits.

x_3 = Percentage of total funds to be invested in equity shares.

x_4 = Percentage of total funds to be invested in time deposits.

x_5 = Percentage of total funds to be invested in indira vikas patra

x_6 = Percentage of total funds to be invested in real estate

Example 11. Round –the –clock Ltd., a departmental store has the following daily requirement for staff.

Period	**Time (24 hrs a day)**	**Minimum staff requirement**
1	**6 am -- 10 am**	**12**
2	**10 am – 2 pm**	**17**
3	**2 pm – 6 pm**	**25**
4	**6 pm – 10 pm**	**18**
5	**10 pm – 2 am**	**20**
6	**2 am – 6 am**	**16**

Staff reports to the store at the start of each period & works for 8 consecutive hours. The departmental store wants to find out the minimal number of staff to be employed so that there will be sufficient number of employees available for each period.

You are required to formulate the problem as a linear programming model giving clearly the constraints & the objective function.

Solution $x_1, x_2, x_3, x_4, x_5, x_6$ be the number of staff members required to be present in the store for the corresponding 1 to 6 periods.

Proceeding in the usual manner:

The objective function can be expressed as under:

Minimize $Z = x_1, x_2, x_3, x_4, x_5, x_6$

(Since objective is to minimize the no. of staff)

Constraint equation: Since each person has to work for 8 consecutive hours, the X_1 employee who are employed during the Ist period shall still be on duty when the IInd period starts. Thus, during the II period there will be $x_1 + x_2$ employees.

Since the minimal number of staff required during the 2nd period is given to be 17, therefore we must have the first constraint equation as: $x_1 + x_2 \geq 17$

like wise the remaining constraint equations can be written as :

$x_2 + x_3 \geq 25$
$x_3 + x4 \geq 18$
$x_4 + x_5 \geq 20$
$x_5 + x_6 \geq 16$ and
$x_{16} + x_1 \geq 2$
where $X_1, X_2 \ldots\ldots\ldots \geq 0$

Example12. The management of Quality Toys wants to determine the number of advertisement to be placed in three monthly magazines M_1 , M_2 & M_3 the primary objective for advertising is to maximize the total exposure to the customers & the potential buyers of its high quality & safe toys. Percentages of readers for each of the 3 magazines M_1 , M_2 & M_3 are know with the help of a readership survey.

The following information is provided.

	Magazines		
Readers (in lakh)	2	1.60	1.40
Principal buyer	10%	10%	5%
Cost / advertisement(Rs.)	5000	4000	3000

The maximum advertisement budget is Rs. 10 lakhs. The management has already decided that magazine M_1 should not have more than 12 advertisements & that M_2 & M_3 can each have at least 2 advertisements. Formulate the above as a LP model.
(exposure in a magazine = No. of advertisement placed X no. of principal buyers)

Ans. Let X_1 , X_2 , $\mathbf{X_3}$ denoted the required no. of advertisements in magazines M_1 , M_2 & M_3 respectively.

Step 1. the total exposure of principal buyers of the magazine is:

$Z = (10\% \text{ of } 200000)\ X_1 + (10\% \text{ of } 160000)\ X_2 + (5\% \text{ of } 140000)\ X_3 = 20000\ X_1 + 16000\ X_2 + 7000\ X_3$

Step 2. list down the constrain equations:

$X_1 \leq 12$ 9since M_1 can not have more than 12 advertisements)

$X_2 \geq 2$ } since M_1 M_2 can have at least 2 advertisements
$X_3 \geq 2$

Step 3. present as LP model i.e., $Z = 20000 X_1 + 16000 X_2 + 7000 X_3$
Maximize (Exposures are ot be maximized)
Subject to : $X_1 \leq 12$
$X_2 \leq 2$
$X_3 \leq 2$
Further X_1 , X_2 **,** $\mathbf{X_3} \geq 0$

Example 13. Precise manufacturing works produces a product each unit of which consists of 10 units of component A & 8 unit of component. B Both these components are manufactured form two different raw materials of which 100 units & 80 unit are available respectively. The manufacture of both these components A & B is done in three distinct departments. The following results pertaining to both the components are given:

Department	Input per cycle		Output per cycle	
	Raw material	Raw material	Component	Component
	X	Y	A	B
P	18	16	17	15
Q	15	19	16	19
R	13	18	18	14

Formulate the given problem as a linear programming model so as to find out the optimal m=number of production runs for each of the three departments that will maximize the total number of complete units of the final product.

Let X_1 , X_2 **,** $\mathbf{X_3}$ respect the no. of production runs for the three departments P,Q,& R respectively
Step 1. Write the objective function

The objective is to maximize the total no. of units of final product. Since each unit of product requires 10 units of component A& 8 units of component B, there fore the maximum number of units of the final products can not exceed the smaller value of :
$(17 X_1 + \mathbf{16}X_2 + \mathbf{18\ X_3})/10$
(i.e. the np. Of units of component A produced by different departments divided by the no. units of A required for the final product)
also $15X_1 + 19 X_2 + \mathbf{14X_3 / 8}$
(i.e. the no of units of component B manufactured by various departments divided by the no. of units of B required for the final product)
the objective function hence becomes:
Maximize

$$Z = \text{Minimum of} \left(\frac{17 X_1 + 16X_2 + 18 X_3}{10}, \frac{15 X_1 + 19X_2 + 14 X_3}{8} \right)$$

Step II. Give the constraint equations:
Raw material constraints:
$18x_1 + 15x_2 + 13x_3 \leq 100$ (Raw material : X)

$16x_1+19x_2+18x_3 \leq 80$ (Raw material : Y)

Subject to $x_1, x_2, x_3 \geq 0$

Hence, the model can be formulated.

Example 14. Unique Ltd. Is contemplating investment expenditure for its plant evaluation & modernization (R& X). Various proposals are lying with the unique Ltd. Has generated the following data which would be utilized in evaluating & selecting the best set of proposals.

Option Definition	Expenditure (Rs. .000)		Engineering required	
	Ist year	IInd Year	Value (Rs. '000)	Hours ('00)
1. Renovate	22	--	7	5
Assembly	9.2	2	9.20	8
2. New assembly	2	7	6	4
3. New machinery	--	1	8	.30
4. Renovate shops	6	7	14.50	9
5. Process materials	6	1	7	4
6. New process	19.	0	4.50	.50
7. New storage facility	50	2		3
	8	2		--
		--		
		3.		
		20		

Following are the budgetary constraints:

Expenditure for Ist year	:	4.00 lacs
Expenditure for IInd year	:	14.40 lacs
Total engineering hours	:	25,000 hours

The prevailing situation necessitates that a new or modernized shop floor be provided. The machinery for production line is applicable only to the new shop floor. The management does not desire to opt for building or buying of raw material processing facilities. Formulate the problem so as to for maximize the total gains to the company i.e. the engineering value.

Assign the following relation to parameter x_1

i.e., Let $xi = \begin{cases} 1, \text{ if project I is to be under taken} \\ 0, \text{ otherwise} \end{cases}$

Objective function

Maximize

$Z = 7x_1 + 9.50x_2 + 6x_3 + 8x_4 + 14 - 50x_5 + 7x_6 + 4.50x_7$

(Since value of engineering i. e money is to be maximized

Constraints:

(i) The total expenditure in the first year should not exceed Rs 4 lacs i.e.;

$22x_1 + 9.50x_2 + 0x_3 + 6x_4 + 6x_5 + 19.50x_6 + 8x_7 \leq 400$

(ii) The total expenditure in the second year should not exceed 14.40 lacs i.e.;

$0x_1 + 27x_2 + 17x_3 + 10x_4 + 22x_5 + 0x_6 + 3.2x_7 \leq 1440$

(iii) The total number of hours required for engineering cannot exceed the maximum of 25,000 hrs. i.e.;

$5x_1 + 8x_2 + 4.30x_3 + 9x_4 + 4.50x_5 + 3x_6 + 30x_7 \leq 250$

(iv) Both the options or new and modernized shop floor can not be taken together. However, at least one option has to be exercised i.e;

$x_1 + x_2 = 1$

(v) Option 3 may be exercised only if option 2 is exercised i.e.;

$x_5 \geq x_2$

(vii) $xi \geq 0$ or for all i.

Based on the above constraints & the objective function, a linear programming model may be constructed.